Three Dimensional Geometry

Question1

If the line joining the points $\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ intersects the plane passing through the points $2\hat{\mathbf{i}} - \hat{\mathbf{j}}, 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\hat{\mathbf{k}} - 2\hat{\mathbf{i}}$ at $\mathbf{r}$, then $\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) =$

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Options:

A.

15

B.

5

C.

3

D.

7

Answer: A

Solution:

Step 1: Find the Direction Vector of the Line

The line joins the points $\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ (which is (1, 2, 0)) and $\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ (which is (0, 1, -2)).

So, the direction vector is found by subtracting these: $(0 - 1, 1 - 2, -2 - 0) = (-1, -1, -2)$.

Step 2: Write the Equation of the Line

The equation of the line is: $(x, y, z) = (1, 2, 0) + t(-1, -1, -2)$.

This gives: $x = 1 - t$, $y = 2 - t$, $z = -2t$.

Step 3: Find the Equation of the Plane

The plane goes through $2\hat{\mathbf{i}} - \hat{\mathbf{j}}$ (2, -1, 0), $2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ (0, 2, 3), and $\hat{\mathbf{k}} - 2\hat{\mathbf{i}}$ (-2, 0, 1).

Step 4: Get Two Vectors in the Plane

Let $P = (2, -1, 0)$, $Q = (0, 2, 3)$, $R = (-2, 0, 1)$. Compute:$PQ = Q - P = (-2, 3, 3)$, $PR = R - P = (-4, 1, 1)$.

Step 5: Find the Normal Vector by Cross Product

The normal vector of the plane is $PQ \times PR$:

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -2 & 3 & 3 \\ -4 & 1 & 1 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(3 \cdot 1 - 3 \cdot 1) - \hat{\mathbf{j}}(-2 \cdot 1 - 3 \cdot -4) + \hat{\mathbf{k}}(-2 \cdot 1 - 3 \cdot -4)$$
$$= \hat{\mathbf{i}}(0) - \hat{\mathbf{j}}(10) + \hat{\mathbf{k}}(10)$$
$$= -10\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$$

Step 6: Write Equation of the Plane

The normal vector is $(0, -10, 10)$. The plane passes through $P = (2, -1, 0)$, so the equation is: $0(x - 2) - 10(y + 1) + 10(z - 0) = 0$.

This simplifies to $-10(y + 1) + 10z = 0$, or $z - y = 1$.

Step 7: Find Where the Line Meets the Plane

Plug the line equations $y = 2 - t$, $z = -2t$ into the plane equation $z - y = 1$:

$-2t - (2 - t) = 1$

$-2t - 2 + t = 1$

$-t - 2 = 1$

$-t = 3$

$t = -3$

Step 8: Find the Intersection Point

Put $t = -3$ into the line equations:

$x = 1 - (-3) = 4$

$y = 2 - (-3) = 5$

$z = -2 \times (-3) = 6$

So, the point $\mathbf{r} = (4, 5, 6)$.

Step 9: Find $\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

This is $4 \times 1 + 5 \times 1 + 6 \times 1 = 15$.

Question2

The vector equation of a plane passing through the line of intersection of the planes $\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{k}}) = 3, \mathbf{r} \cdot (2\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ and the point $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ is

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Options:

A.

$\mathbf{r} \cdot (\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) = 13$

B.

$\mathbf{r} \cdot (\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 18$

C.

$\mathbf{r} \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) = 8$

D.

$\mathbf{r} \cdot (\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 23$

Answer: D

Solution:

A plane passing through the line of intersection of two planes

$\mathbf{r} \cdot \mathbf{n}_1 = d_1$ and $\mathbf{r} \cdot \mathbf{n}_2 = d_2$
is $\mathbf{r} \cdot (\mathbf{n}_1 + \lambda \mathbf{n}_2) = d_1 + \lambda d_2$
Here, $\mathbf{n}_1 = \hat{\mathbf{i}} - 2\hat{\mathbf{k}}, \mathbf{n}_2 = 2\hat{\mathbf{j}} + \hat{\mathbf{k}}, d_1 = 3$
and $d_2 = 5$

$$\therefore \mathbf{r} \cdot ((\hat{\mathbf{i}} - 2\hat{\mathbf{k}}) + \lambda(2\hat{\mathbf{j}} + \hat{\mathbf{k}})) = 3 + 5\lambda$$
$$\Rightarrow \mathbf{r} \cdot (\hat{\mathbf{i}} + 2\lambda\hat{\mathbf{j}} + (-2 + \lambda)\hat{\mathbf{k}}) = 3 + 5\lambda \qquad \ldots (i)$$

put $\mathbf{r} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
then, $1 + 4\lambda + 3(-2 + \lambda) = 3 + 5\lambda$
$\Rightarrow \quad 1 - 6 + 7\lambda = 3 + 5\lambda \Rightarrow \lambda = 4$
Substitute λ in Eq. (i)
$\mathbf{r} \cdot (\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 23$

Question3

The points $A(-1, 2, 3)$, $B(2, -3, 1)$ and $C(3, 1, -2)$

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Options:

A.

are collinear

B.

form an isosceles triangle

C.

form a right-angled triangle

D.

form a scalene triangle

Answer: D

Solution:

$a = (-1, 2, 3), B = (2, -3, 1),$

$C = (3, 1, -2)$
$AB = \sqrt{(2+1)^2 + (-3-2)^2 + (1-3)^2} = \sqrt{37}$
$BC = \sqrt{(3-2)^2 + (1+3)^2 + (-2-1)^2} = \sqrt{26}$

and

$AC = \sqrt{(3+1)^2 + (1-2)^2 + (-2-3)^2} = \sqrt{42}$

$\because AB \neq BC \neq AC$ and it does not follow right angled triangle property.

Hence, A, B, C form a scalene triangle.

Question4

The directions cosines of the line making angles $\frac{\pi}{4}, \frac{\pi}{3}$ and $\theta\left(0<\theta<\frac{\pi}{2}\right)$ respectively with X, Y and Z axes are

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Options:

A.

$\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$

B.

$\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{\sqrt{3}}{2}$

C.

$\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{\sqrt{2}}$

D.

$\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$

Answer: A

Solution:

$\cos \frac{\pi}{4}=\frac{1}{\sqrt{2}}, \cos \frac{\pi}{3}=\frac{1}{2}$

and we know, $l^2+m^2+n^2=1$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Hence, direction cosines are $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$

Question5

If the equation of the plane passing through the point $(3, 2, 5)$ and perpendicular to the planes $2x - 3y + 5z = 7$ and $5x + 2y - 3z = 11$ is $x + by + cz + d = 0$, then $2b + 3c + d =$

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Options:

A.

0

B.

35

C.

1

D.

20

Answer: B

Solution:

Direction ratio of normal to the plane $2x - 3y + 5z = 7$ is $2, -3, 5$ and to $5x + 2y - 3z = 11$ is $\langle 5, 2, -3\rangle$ $\because$ Required plane is perpendicular to both, its normal vector $\langle 1, b, c\rangle$ must be perpendicular to both given normal vectors.

$\therefore \langle 1, b, c\rangle \cdot \langle 2, -3, 5\rangle = 2 - 3b + 5c = 0 \qquad \ldots (i)$

and $\langle 1, b, c\rangle \cdot \langle 5, 2, -3\rangle$

$= 5 + 2b - 3c = 0 \quad \ldots(ii)$

By Eq. (i) and (ii), we get

$b = -31, c = -19$

The equation of plane is $x - 31y - 19z + d = 0$ is passes through $(3, 2, 5)$

$\therefore 3 - 62 - 95 + d = 0 \Rightarrow d = 154$

Hence, $2b + 3c + d = 2(-31) + 3(-19) + 154 = 35$

Question6

Line L_1 passes through the point $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\hat{\mathbf{k}} - \hat{\mathbf{i}}$. Line L_2 passes through the point $\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and is parallel to the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$. If $x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ is the point of intersection of the lines L_1 and L_2, then $(y - x) =$

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Options:

A.

$2z$

B.

$-2z$

C.

z

D.

$-z$

Answer: C

Solution:

$\because L_1$ passes through $\hat{\mathbf{i}}+\hat{\mathbf{j}}$ i.e., $(1,1,0)$ and $k-i$ i.e., $(-1,0,1)$

$\therefore$ Equation of L_1

$$\begin{aligned} & \frac{x-1}{-1-1}=\frac{y-1}{0-1}=\frac{z-0}{1-0} \\ \Rightarrow & \frac{x-1}{-2}=\frac{y-1}{-1}=\frac{z}{1} \\ \Rightarrow & \frac{x-1}{2}=\frac{y-1}{1}=\frac{z}{-1} \end{aligned}$$

any point $(2\lambda+1, \lambda+1, -\lambda)$

Similarly L_2 passes through $(0,1,2)$ and parallel $\mathbf{i}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$\Rightarrow$ Equation of L_2

$\frac{x-0}{1}=\frac{y-1}{1}=\frac{z-2}{1}$

Any point on $L_2 \Rightarrow (\mu, \mu+1, \mu+2)$

$\therefore$ Point of intersection

$$\begin{aligned} & 2\lambda+1=\mu, \lambda+1=\mu+1, -\lambda=\mu+2 \\ \therefore \quad & \lambda=\mu \Rightarrow \lambda=\mu=-1 \end{aligned}$$

$\therefore$ Point of intersection $=(-1,0,1)$

$\therefore y-x=1=z$

Question7

The point in the XY - plane which is equidistant from the points $A(2,0,3), B(0,3,2)$ and $C(0,0,1)$ has the coordinates

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Options:

A.

$(3,2,0)$

B.

$(2,3,0)$

C.

$(2,0,8)$

D.

$(0,3,1)$

Answer: A

Solution:

Let $P(x, y, z)$ be a point that is the same distance from each of the points $A(2,0,3)$, $B(0,3,2)$, and $C(0,0,1)$.

To find P, let's set all the distances equal:

Step 1: Equal Distance from A and B

The distance from P to A is $\sqrt{(x-2)^2+(y-0)^2+(z-3)^2}$.

The distance from P to B is $\sqrt{(x-0)^2+(y-3)^2+(z-2)^2}$.

Set these distances equal (we can remove the square roots since both must be positive):

$(x-2)^2+y^2+(z-3)^2=x^2+(y-3)^2+(z-2)^2$

Simplify Equation 1:

Expand both sides:

Left: $x^2-4x+4+y^2+z^2-6z+9$
Right: $x^2+y^2-6y+9+z^2-4z+4$

Subtract the right side from the left side:

$-4x-6z+4+9-(-6y+9-4z+4)=0$

$-4x-6z+13+6y-9+4z-4=0$

Group like terms:

$-4x+6y-2z=0$

Or, $2x-3y+z=0$ (divide by -2)

This is Equation (i).

Step 2: Equal Distance from B and C

Write their distances equal:

$(x-0)^2+(y-3)^2+(z-2)^2=x^2+y^2+(z-1)^2$

Expand both sides:

Left: $x^2 + y^2 - 6y + 9 + z^2 - 4z + 4$
Right: $x^2 + y^2 + z^2 - 2z + 1$

Subtract the right side from the left side:

$-6y + 9 - 4z + 4 - (-2z + 1) = 0$

$-6y + 13 - 4z + 2z - 1 = 0$

$-6y - 2z + 12 = 0$

Or, $3y + z = 6$ (divide by -2)

This is Equation (ii).

Step 3: Solve Both Equations Together

We now solve these:

Equation (i): $2x - 3y + z = 0$
Equation (ii): $3y + z = 6$

From Equation (ii): $z = 6 - 3y$

Substitute z in Equation (i):
$2x - 3y + (6 - 3y) = 0$
$2x - 6y + 6 = 0$

$2x = 6y - 6$
$x = 3y - 3$

Let's set $y = 2$ (for a possible solution):
$\rightarrow x = 3(2) - 3 = 6 - 3 = 3$
$z = 6 - 3(2) = 6 - 6 = 0$

So, the point P with coordinates $(3, 2, 0)$ fits both equations.

Question8

If the direction ratio of two lines L_1 and L_2 are given by $(1, -2, 2)$ and $(-2, 3, -6)$ respectively, then the direction ratios of the line which is perpendicular to the linesh and L_2 are

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Options:

A.

$(1, -2, 3)$

B.

$(-2, 3, 5)$

C.

$(6, 2, -1)$

D.

$(2, -1, 3)$

Answer: C

Solution:

We are given the direction ratios for L_1 as $1, -2, 2$.

We are also given the direction ratios for L_2 as $-2, 3, -6$.

To find the direction ratios for a line that is perpendicular to both L_1 and L_2, we need to take the cross product of the direction ratios of L_1 and L_2.

Let us use the formula for the cross product. If the direction ratios of the first line are (a_1, b_1, c_1) and of the second line are (a_2, b_2, c_2), then the perpendicular line will have direction ratios:

$(b_1c_2 - c_1b_2,\ c_1a_2 - a_1c_2,\ a_1b_2 - b_1a_2)$

Now substitute the given values:

$(-2 \times -6 - 2 \times 3,\ 2 \times -2 - 1 \times -6,\ 1 \times 3 - (-2) \times -2)$

This becomes:

$(12 - 6,\ -4 + 6,\ 3 - 4)$

Which simplifies to:

$6, 2, -1$

So, the direction ratios for the required line are $6, 2, -1$.

Question9

If the image of the point $A(1, 1, 1)$ with respect to the plane $4x + 2y + 4z + 1 = 0$ is $B(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma =$

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Options:

A.

-2

B.

$-\frac{28}{9}$

C.

$\frac{55}{36}$

D.

$\frac{35}{16}$

Answer: B

Solution:

Step 1: Formula for the image of a point with respect to a plane

If you have a point $A(x_1, y_1, z_1)$ and a plane $ax + by + cz + d = 0$, the image point $B(\alpha, \beta, \gamma)$ is found using the following:

$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{\gamma - z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$

Step 2: Plug in the values

Here, $A(1, 1, 1)$ and the plane is $4x + 2y + 4z + 1 = 0$. So, $a = 4$, $b = 2$, $c = 4$, and $d = 1$.

Calculate $ax_1 + by_1 + cz_1 + d = 4 \times 1 + 2 \times 1 + 4 \times 1 + 1 = 4 + 2 + 4 + 1 = 11$.

Calculate $a^2 + b^2 + c^2 = 4^2 + 2^2 + 4^2 = 16 + 4 + 16 = 36$.

Now, substitute both results into the formula:

$\frac{\alpha - 1}{4} = \frac{\beta - 1}{2} = \frac{\gamma - 1}{4} = \frac{-2 \times 11}{36}$

This simplifies to:

$\frac{\alpha - 1}{4} = \frac{\beta - 1}{2} = \frac{\gamma - 1}{4} = \frac{-22}{36} = \frac{-11}{18}$

Step 3: Solve for the coordinates

For $\alpha : \frac{\alpha - 1}{4} = \frac{-11}{18} \Rightarrow \alpha - 1 = 4 \cdot \frac{-11}{18} = \frac{-44}{18} = \frac{-22}{9} \Rightarrow \alpha = 1 + \frac{-22}{9} = \frac{-13}{9}$

For $\beta : \frac{\beta - 1}{2} = \frac{-11}{18} \Rightarrow \beta - 1 = 2 \cdot \frac{-11}{18} = \frac{-22}{18} = \frac{-11}{9} \Rightarrow \beta = 1 + \frac{-11}{9} = \frac{-2}{9}$

For $\gamma : \frac{\gamma - 1}{4} = \frac{-11}{18} \Rightarrow \gamma - 1 = 4 \cdot \frac{-11}{18} = \frac{-44}{18} = \frac{-22}{9} \Rightarrow \gamma = 1 + \frac{-22}{9} = \frac{-13}{9}$

Step 4: Add the coordinates

$\alpha+\beta+\gamma=\frac{-13}{9}+\frac{-2}{9}+\frac{-13}{9}=\frac{-13-2-13}{9}=\frac{-28}{9}$

Question10

Assertion (A) For the lines $\mathbf{r}=\mathbf{a}+t\mathbf{b}$ and $\mathbf{r}=\mathbf{p}+s\mathbf{q}$, if $(\mathbf{a}-\mathbf{p})\cdot(\mathbf{b}\times\mathbf{q})\neq 0$, then the two lines are coplanar.

Reason (R)$|(\mathbf{a}-\mathbf{p})\cdot(\mathbf{b}\times\mathbf{q})|$ is $|\mathbf{b}\times\mathbf{q}|$ times the shortest distance between the lines $\mathbf{r}=\mathbf{a}+t\mathbf{b}$ and $\mathbf{r}=\mathbf{p}+s\mathbf{q}$.

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Options:

A.

(A) is true, (R) is true and (R) is correct explanation to (A)

B.

(A) is true, (R) is true and (R) is not the correct explanation to (A)

C.

(A) is true, (R) is false

D.

(A) is false, (R) is true

Answer: D

Solution:

Assertion Given lines $\mathbf{r}=\mathbf{a}+t\mathbf{b}$ and $\mathbf{r}=\mathbf{p}+s\mathbf{q}$

We know that the lines are coplanar if $(\mathbf{a}-\mathbf{p})\cdot(\mathbf{b}\times\mathbf{q})=0$

Thus, the assertion is wrong.

Reason The shortest distance between two skew lines is

$$d=\frac{|(\mathbf{a}-\mathbf{p})\cdot(\mathbf{b}\times\mathbf{q})|}{|\mathbf{b}\times\mathbf{q}|}$$

$$\Rightarrow \quad d|\mathbf{b}\times\mathbf{q}|=|(\mathbf{a}-\mathbf{p})\cdot(\mathbf{b}\times\mathbf{q})|$$

Thus, the reason is

So, A is false, but R is true.

Question11

The locus of a point at which the line joining the points $(-3,1,2),(1,-2,4)$ subtends a right angle, is

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Options:

A.

$x^2+y^2+z^2+2x+y-6z-3=0$

B.

$x^2+y^2+z^2+2x-y-6z+3=0$

C.

$x^2+y^2+z^2+2x+y-6z+3=0$

D.

$x^2+y^2+z^2-2x+y-6z+3=0$

Answer: C

Solution:

Let the two fixed points be $A(-3,1,2)$ and $B(1,-2,4)$.

Let $P(x,y,z)$ be the point at which the line segment AB subtends a right angle.

Then, $\angle APB=90^\circ$

$$\Rightarrow \quad \mathbf{PA}\cdot\mathbf{PB}=0 \qquad \ldots(i)$$

Now, $\mathbf{PA}=(-3-x,1-y,2-z)$

$\mathbf{PB} = (1 - x, -2 - y, 4 - z)$

So, $\mathbf{PA} \cdot \mathbf{PB} = (-3 - x)(1 - x) + (1 - y)(-2 - y) + (2 - z)(4 - z) = 0$

$$\Rightarrow \quad -3 + 3x - x + x^2 - 2 - y + 2y + y^2 + 8 - 2z - 4z + z^2 = 0$$
$$\Rightarrow \quad x^2 + 2x + y^2 + y + z^2 - 6z + 3 = 0$$
$$\Rightarrow \quad x^2 + y^2 + z^2 + 2x + y - 6z + 3 = 0$$

Question12

If $A(1, 2, 3), B(2, 3, -1), C(3, -1, -2)$ are the vertices of a $\triangle ABC$, then the direction ratios of the bisector of $\angle ABC$ are

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Options:

A.

$(4, 1, 1)$

B.

$(3, 5, 2)$

C.

$(1, 4, 1)$

D.

$(2, -3, -5)$

Answer: D

Solution:

Given, vertices of $\triangle ABC$ are $A(1, 2, 3), B(2, 3, -1)$ and $C(3, -1, -2)$

Now, $AB = B - A = (2, 3, -1) - (1, 2, 3) = (1, 1, -4)$

Now,

$\mathbf{CB} = (3, -1, -2) - (2, 3, -1) = (1, -4, -1)$

$$So,\ |\mathbf{AB}| = \sqrt{1^2+1^2+(-4)^2}$$
$$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$$
$$|\mathbf{CB}| = \sqrt{1^2+(-4)^2+(-1)^2}$$
$$= \sqrt{18} = 3\sqrt{2}$$

So, d.c. of $\mathbf{AB} = \frac{\mathbf{AB}}{|\mathbf{AB}|} = \frac{(1,1,-4)}{3\sqrt{2}}$

$$= \left(\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{-4}{3\sqrt{2}}\right)$$

And d.c. of $\mathbf{CB} = \frac{\mathbf{CB}}{|\mathbf{CB}|} = \frac{(1,-4,-1)}{3\sqrt{2}}$

$$= \left(\frac{1}{3\sqrt{2}}, \frac{-4}{3\sqrt{2}}, \frac{-1}{3\sqrt{2}}\right)$$

$\therefore$ d.c.'s of the internal bisector of $\angle ABC$ are

$$\left(\frac{1}{3\sqrt{2}}+\frac{1}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}-\frac{4}{3\sqrt{2}}, \frac{-4}{3\sqrt{2}}-\frac{1}{3\sqrt{2}}\right)$$
$$= \left(\frac{2}{3\sqrt{2}}, \frac{-3}{3\sqrt{2}}, \frac{-5}{3\sqrt{2}}\right)$$

So, the D.c.'s are $\langle 2, -3, -5 \rangle$.

Question13

Let $A = (2, 0, -1), B = (1, -2, 0), C = (1, 2, -1)$ and $D = (0, -1, -2)$ be four points.

If θ is the acute angle between the plane determined by A, B, C and the plane determined by A, C, D, then $\tan\theta =$

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Options:

A.

$\sqrt{\frac{14}{5}}$

B.

$\frac{3}{\sqrt{14}}$

C.

$\frac{3}{\sqrt{5}}$

D.

$\frac{\sqrt{5}}{3}$

Answer: C

Solution:

Given points are $A(2,0,-1), B(1,-2,0), C(1,2,-1)$ and $D(0,-1,-2)$.

So, $$\begin{aligned} \mathbf{AB} &= B - A = (1,-2,0)-(2,0,-1) \\ &= (-1,-2,+1) \\ \mathbf{AC} &= C - A = (1,2,-1)-(2,0,-1) \\ &= (-1,2,0) \end{aligned}$$

Now, normal vector

$$\mathbf{n}_1 = \mathbf{AB} \times \mathbf{AC} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & -2 & 1 \\ -1 & 2 & 0 \end{vmatrix}$$

$$\begin{aligned} &= (0-2)\hat{\mathbf{i}} - (0+1)\hat{\mathbf{j}} + (-2-2)\hat{\mathbf{k}} \\ &= -2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 4\hat{\mathbf{k}} = (-2,-1,-4) \\ &\text{Now, } \mathbf{AD} = D - A = (0,-1,-2)-(2,0,-1) \\ &= (-2,-1,-1) \end{aligned}$$

So, normal vector $n_2 = \mathbf{AC} \times \mathbf{AD}$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -1 & 2 & 0 \\ -2 & -1 & -1 \end{vmatrix}$$

$$\begin{aligned} &= (-2-0)\hat{\mathbf{i}} - (1+0)\hat{\mathbf{j}} + (1+4)\hat{\mathbf{k}} \\ &= -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}} = (-2,-1,5) \end{aligned}$$

$$\text{So, } \mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -2 & -1 & -4 \\ -2 & -1 & 5 \end{vmatrix}$$

$$\begin{aligned} = (-5-4)\hat{\mathbf{i}} - (-10-8)\hat{\mathbf{j}} + (2-2)\hat{\mathbf{k}} \\ = -9\hat{\mathbf{i}} + 18\hat{\mathbf{j}} + 0\hat{\mathbf{k}} \\ (-9,18,10) \end{aligned}$$

And $\mathbf{n}_1 \cdot \mathbf{n}_2 = (-2,-1,-4) \cdot (-2,-1,5)$

$= 4 + 1 - 20 = -15$

So, the tangent of the angle between the two planes

$$\tan\theta = \frac{|\mathbf{n}_1 \times \mathbf{n}_2|}{|\mathbf{n}_1 \cdot \mathbf{n}_2|} = \frac{\sqrt{(-9)^2 + (18)^2 + 0^2}}{|-15|}$$
$$= \frac{\sqrt{81 + 324 + 0}}{|-15|} = \frac{\sqrt{405}}{15}$$
$$= \frac{9\sqrt{5}}{15} = \frac{3\sqrt{5}}{5} = \frac{3}{\sqrt{5}}$$

So, the tangent of the acute angle between the two planes is $\frac{3}{\sqrt{5}}$.

Question14

If $A(0,1,2)$, $B(2,-1,3)$ and $C(1,-3,1)$ are the vertices of a triangle, then the distance between its circumcentre and orthocentre is

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Options:

A.

$\frac{3}{\sqrt{2}}$

B.

$\frac{3}{2}$

C.

3

D.

$\frac{9}{2}$

Answer: A

Solution:

$A(0,1,2), B(2,-1,3), C(1,-3,1)$

$\because \quad \mathbf{BA} =< -2, 2, -1 \geqslant$
$\quad \mathbf{BC} =< -1, -2, -2 >$

$\because \quad \mathbf{BA} \cdot \mathbf{BC} = (-2)(-1) + 2(-2)$
$\quad +(-1)(-2) = 0$

So, $\triangle ABC$ is right angled at B.

$\therefore$ Orthocenter (H) is at point $B = (2,-1,3)$ and circumcenter

$(O) = \left(\frac{0+1}{2}, \frac{-3+1}{2}, \frac{1+2}{2}\right) = \left(\frac{1}{2}, -1, \frac{3}{2}\right)$

Hence,

$$OH = \sqrt{\left(2-\frac{1}{2}\right)^2 + (-1+1)^2 + \left(3-\frac{3}{2}\right)^2}$$
$$= \sqrt{\frac{9}{4}+\frac{9}{4}} = \frac{3}{\sqrt{2}}$$

Question15

If the direction cosines of two lines satisfy the equations $l-2m+n=0, lm+10mn-2nl=0$ and θ is the angle between the lines, then $\cos\theta =$

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Options:

A.

$\frac{\pi}{6}$

B.

$\frac{8}{\sqrt{70}}$

C.

$\frac{\pi}{3}$

D.

$\frac{20}{3\sqrt{70}}$

Answer: B

Solution:

$\because l-2m+n=0 \Rightarrow l=2m-n$

Substitute in $lm+10mn-2nl=0$

$$\begin{aligned}&\Rightarrow (2m-n)m+100mn-2n(2m-n)=0\\&\Rightarrow 2m^2+5mn+2n^2=0\\&\Rightarrow m=\frac{-5n\pm 3n}{4}=\frac{-n}{2},-2n\end{aligned}$$

If $m=\frac{-n}{2}, l=-2n$

Then, direction ratios of are

$1:m:n=-2n:\frac{n}{2}:n=-4:-1:2$

and direction cosine are

$$\begin{aligned}(-4k)^2+(-k)^2+(2k)^2=1 \Rightarrow k=\tfrac{1}{\sqrt{21}}\\ =\left(\tfrac{-4}{\sqrt{21}},\tfrac{-1}{\sqrt{21}},\tfrac{2}{\sqrt{21}}\right)\end{aligned}$$

If $m=-2n$, then $l=-5n$ and DR's are $l:m:n=-5:-2:1$ and DC's are $(-5k)^2+(-2k)^2+k^2=1$

$\Rightarrow \quad k=\frac{1}{\sqrt{30}}=\left(\frac{-5}{\sqrt{30}},\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}}\right)$

Therefore angle between given lines are

$$\cos\theta=\left|l_1l_2+m_1m_2+n_1n_2\right|$$
$$=\left|\begin{array}{l}\frac{-4}{\sqrt{21}}\times\left(\frac{-5}{\sqrt{30}}\right)+\left(\frac{-1}{\sqrt{21}}\right)\\ \left(\frac{-2}{\sqrt{30}}\right)+\left(\frac{2}{\sqrt{21}}\right)\left(\frac{1}{\sqrt{30}}\right)\end{array}\right|$$
$$\begin{aligned}&=\frac{24}{\sqrt{630}}=\frac{24}{3\sqrt{70}}\\&=\frac{8}{\sqrt{70}}\end{aligned}$$

Question16

If $(2,-1,3)$ is the foot of the perpendicular drawn from the origin $(0,0,0)$ to a plane, then the equation of that plane is

Options:

A.

$2x + y - 3z + 6 = 0$

B.

$2x - y + 3z - 14 = 0$

C.

$2x - y + 3z - 13 = 0$

D.

$2x + y + 3z - 10 = 0$

Answer: B

Solution:

As given, the vector from the origin to the point $(2, -1, 3)$ is normal to plane.

$\therefore \quad \mathbf{n} = < 2, -1, 3 >$

Then, general equation of plane

$2x - y + 3z = d$

$\because \quad (2, -1, 3)$ lies on the plane.

$\therefore \quad 2(2) - (-1) + 3 \times 3 = d \Rightarrow d = 14$

Hence, equation of the plane

$2x - y + 3z - 14 = 0$

Question17

If $A(2, -1, 1)$, $B(2, 5, 1)$ and $C(0, -2, 3)$ are the vertices of a triangle. If D is the point of intersection of the side BC and the internal angular bisector of angle A, then $AD =$

Options:

A.

$\frac{5}{\sqrt{7}}$

B.

$\frac{3}{\sqrt{2}}$

C.

$\frac{\sqrt{3}}{2}$

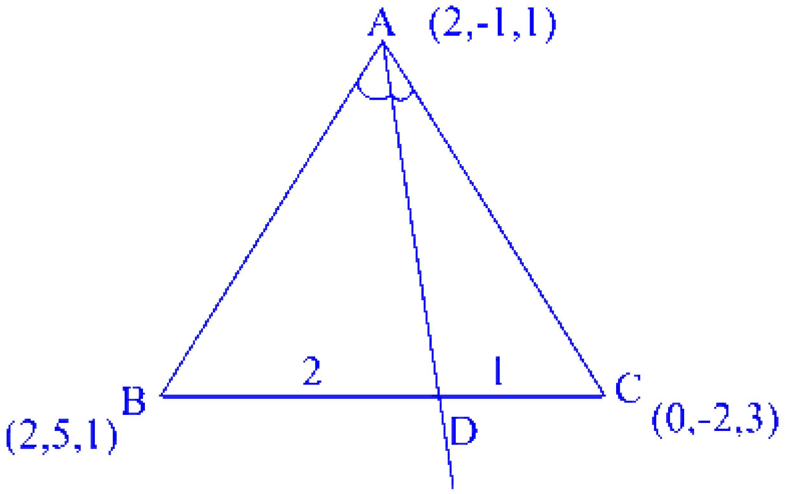
D.

$\frac{4}{\sqrt{3}}$

Answer: D

Solution:

Given, $A=(2,-1,1), B=(2,5,1), C=(0,-2,3)$



Using angle bisector theorem,

$$\frac{AB}{AC}=\frac{BD}{DC}$$

$$\Rightarrow \frac{\sqrt{(2-2)^2+(5-(-1))^2+(1-1)^2}}{\sqrt{(0-2)^2+(-2+1)^2+(3-1)^2}} = \frac{BD}{DC}$$
$$\Rightarrow \frac{BD}{DC} = \frac{6}{3} = \frac{2}{1}$$
$$\therefore BD : DC = 2 : 1$$

Now, we find coordinates of D by the help of section formula,

$$D = \left(\frac{2\times0+1\times2}{2+1}, \quad \frac{2\times(-2)+1\times5}{2+1}, \quad \frac{2\times3+1\times1}{2+1}\right)$$
$$= \left(\frac{2}{3}, \frac{1}{3}, \frac{7}{3}\right)$$

Hence,

$$AD = \sqrt{\left(2-\frac{2}{3}\right)^2 + \left(-1-\frac{1}{3}\right)^2 + \left(1-\frac{7}{3}\right)^2}$$
$$= \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{-4}{3}\right)^2 + \left(\frac{-4}{3}\right)^2}$$
$$= \sqrt{\frac{48}{9}} = \sqrt{\frac{16}{3}} = \frac{4}{\sqrt{3}}$$

Question18

A plane π given by $ax + by + 11z + d = 0$ is perpendicular to the planes $2x - 3y + z = 4$, $3x + y - z = 5$ and the perpendicular distance from the origin to the plane π is $\sqrt{6}$ units. If all the intercepts made by the plane π on the coordinate axes are positive, then $d =$

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Options:

A.

ab

B.

$-2ab$

C.

$4ab$

D.

$-3ab$

Answer: D

Solution:

Given equation of plane π is

$ax + by + 11z + d = 0 \qquad \ldots(i)$

Now, the normal vector of π is

$\mathbf{n}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + 11\hat{\mathbf{k}} = \langle a, b, 11\rangle$

Normal of first plane:

$\mathbf{n}_2 = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 1 \cdot \hat{\mathbf{k}} = \langle 2, -3, 1\rangle$

Normal of second plane:

$\mathbf{n}_3 = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}} = \langle 3, 1, -1\rangle$

According to question, if plane π is perpendicular to both then its normal vector is perpendicular to the normals of the two planes.

So, $\mathbf{n}_1 \cdot \mathbf{n}_2 = 0 \Rightarrow 2a - 3b + 11 = 0 \qquad \ldots(ii)$

$\mathbf{n}_1 \cdot \mathbf{n}_3 = 0 \Rightarrow 3a + b - 11 = 0 \qquad \ldots(iii)$

On solving Eqs. (ii) and (iii) we get,

$$a = 2 \text{ and } b = 11 - 3a = 11 - 6 = 5$$
$$\therefore < a, b, 11 > = < 2, 5, 11 >$$

$\therefore$ Equation of plane will become,

$2x + 5y + 11z + d = 0 \qquad \ldots(iv)$

Now, distance from origin $(0, 0, 0)$ to the plane (iv) is

$$\frac{|d|}{\sqrt{2^2 + 5^2 + 11^2}} = \sqrt{6}$$
$$\Rightarrow \quad |d| = \sqrt{6} + \sqrt{150} = 30$$
$$\Rightarrow \quad d = \pm 30$$

(given)

As question demand is all intercepts (i.e. $x, y \times z$ intercepts) are positive.

So, d must be negative because x-intercept $= \frac{-d}{2} > 0 \Rightarrow d < 0$$y$-intercept $= -\frac{d}{5} > 0 \Rightarrow d < 0$

z-intercept $= \frac{-d}{11} > 0 \Rightarrow d < 0$

Hence, $d = -30 = -3 \times 2 \times 5 = -3ab$

Question19

For a positive real number p, if the perpendicular distance from a point $-\hat{\mathbf{i}} + p\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ to the plane $\mathbf{r} \cdot (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) = 7$ is 6 units, then $p =$

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Options:

A.

$\frac{4}{5}$

B.

$\frac{5}{6}$

C.

6

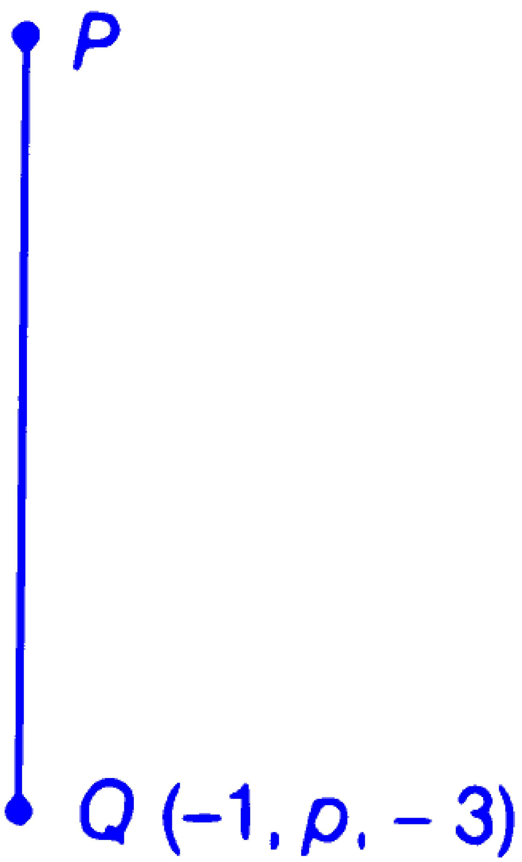
D.

5

Answer: D

Solution:

Point $(-\hat{\mathbf{i}} + p\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \Rightarrow$ Point
(-1, p, -3)
Plane r $\cdot (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) = 7$
$\Rightarrow 2x - 3y + 6z - 7 = 0$



$$PQ = \frac{|-2-3p-18-7|}{\sqrt{2^2+(-3)^2+6^2}}$$

$$6 = \frac{|-3p-27|}{\sqrt{49}}$$

$$42 = |-3p-27|$$

$$\Rightarrow (-3p-27) = \pm 42$$

$$\Rightarrow -3p-27 = 42$$

$$\Rightarrow -3p = 69$$

$$\Rightarrow \quad p = -23$$

$$\Rightarrow -3p-27 = -42$$

$$\Rightarrow -3p = -15$$

$$p = 5$$

Question20

If $Q(\alpha, \beta, \gamma)$ is the harmonic conjugate of the point $P(0, -7, 1)$ with respect to the line segment joining the points $(2, -5, 3)$ and

$(-1,-8,0)$, **then** $\alpha-\beta+\gamma=$

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Options:

A.

4

B.

3

C.

2

D.

1

Answer: A

Solution:

Let P divide AB in the ratio $m:n$.

$$0=\frac{2n-m}{m+n},-7=\frac{-5n-8m}{m+n},1=\frac{3n}{m+n}$$
$$\Rightarrow 2n-m=0\Rightarrow m=2n$$

$\Rightarrow P$ divides AB in the ratio $2:1$

Since Q is the hormonic conjugate of P it divides AB in the ratio $-2:1$

Using the section formula with the ratio -2:1

$$\alpha=\frac{2-2\times(-1)}{1-2}=-4$$
$$\beta=\frac{-5-2(-8)}{1-2}=-11$$
$$\gamma=\frac{3-2(0)}{1-2}=\frac{3}{-1}=-3$$

Thus, $Q=(-4,-11,-3)$

$\Rightarrow\quad \alpha-\beta+\gamma=-4+11-3=4$

Question21

On a line with direction cosines $l, m, n, A\left(x_1, y_1, z_1\right)$ is a fixed point. If $B=\left(x_1+4 k l, y_1+4 k m, z_1+4 k n\right)$ and $C=\left(x_1+k l, y_1+k m, z_1+k n\right)(k>0)$, then the ratio in which the point B divides the line segment joining A and C is

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Options:

A.

$1: 2$

B.

$1:-4$

C.

$4:-3$

D.

$4: 3$

Answer: C

Solution:

Let B divide AC in the ratio λ: 1 The coordinates of B are given by

$$\begin{aligned}
x_B & =\frac{\lambda\left(x_1+k l\right)+1 x_1}{\lambda+1} \\
y_B & =\frac{\lambda\left(y_1+k m\right)+1 y_1}{\lambda+1} \\
Z_B & =\frac{\lambda\left(z_1+k n\right)+1 z_1}{\lambda+1} \\
\Rightarrow x_1+4 k l & =\frac{\lambda\left(x_1+k l\right)+x_1}{\lambda+1} \\
\Rightarrow y_1+4 k m & =\frac{\lambda\left(y_1+k m\right)+y_1}{\lambda+1} \\
\Rightarrow z_1+4 k n & =\frac{\lambda\left(z_1+k n\right)+z_1}{\lambda+1}
\end{aligned}$$

Now, $(x_1 + 4kl)(\lambda + 1) = \lambda(x_1 + kl) + x_1$

$$\Rightarrow x_1\lambda + x_1 + 4kl\lambda + 4kl = x_1\lambda + kl\lambda + x_1$$
$$\Rightarrow \quad 4kl\lambda + 4kl = kl\lambda$$
$$\Rightarrow \quad \lambda = \frac{4kl}{-3kl} = \frac{4}{-3} \Rightarrow \text{ Ratio } = 4 : -3$$

Question22

If the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$ makes an angle α with the positive X-axis, then $\cos\alpha =$

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Options:

A.

$\frac{1}{\sqrt{3}}$

B.

$\frac{1}{\sqrt{2}}$

C.

$\frac{1}{2}$

D.

$\frac{\sqrt{3}}{2}$

Answer: A

Solution:

$(2,3,1)(1,3,2)$

$$\mathbf{n}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2 & 3 & 1\\ 1 & 3 & 2\end{vmatrix}$$
$$=\hat{\mathbf{i}}(6-3)-\hat{\mathbf{j}}(4-1)+\hat{\mathbf{k}}(6-3)$$
$$=3\hat{\mathbf{i}}-3\hat{\mathbf{j}}+3\hat{\mathbf{k}}$$

Since, lines of intersection of the given planes makes an angle α with positive X-axis.

$\therefore \cos\alpha=\frac{3}{\sqrt{3^2+(-3)^2+3^2}}=\frac{3}{3\sqrt{3}}=\frac{1}{\sqrt{3}}$

Question23

$\hat{\mathbf{i}}-2\hat{\mathbf{j}}$ is a point on the line parallel to the vector $2\hat{\mathbf{i}}+\hat{\mathbf{k}}$. If $\hat{\mathbf{i}}+2\hat{\mathbf{j}}$ is a point on the plane parallel to the vectors $2\hat{\mathbf{j}}-\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+2\hat{\mathbf{k}}$, then the point of intersection of the line and the plane is

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Options:

A.

$-\frac{1}{3}(\hat{\mathbf{i}}+6\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

B.

$\frac{1}{3}(\hat{\mathbf{i}}+6\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

C.

$-\frac{1}{3}(\hat{\mathbf{i}}-6\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

D.

$\frac{1}{3}(\hat{\mathbf{i}}-6\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

Answer: A

Solution:

Equation of line,

$r=(\hat{\mathbf{i}}-2\hat{\mathbf{j}})+\lambda(2\hat{\mathbf{i}}+\hat{\mathbf{k}}) \qquad \ldots(i)$

Plane passes through $\hat{\mathbf{i}}+2\hat{\mathbf{j}}$ and is parallel to the vectors $\mathbf{v}_1=2\hat{\mathbf{j}}-\hat{\mathbf{k}}$,

$\mathbf{v}_2=\hat{\mathbf{i}}+2\hat{\mathbf{k}}$

Any point on the plane can be written as

$\mathbf{r}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+x(2\hat{\mathbf{j}}-\hat{\mathbf{k}})+v(\hat{\mathbf{i}}+2\hat{\mathbf{k}}) \qquad \ldots(ii)$

From Eqs. (i) and (ii),

$$1+2\lambda=1+V\Rightarrow 2\lambda=V \qquad \ldots(iii)$$
$$x=-2 \qquad \ldots(iv)$$
$$\lambda=-x+2V \qquad \ldots(v)$$

We get $\lambda=-\frac{2}{3}$

$$\mathbf{r}=(\hat{\mathbf{i}}-2\hat{\mathbf{j}})-\frac{2}{3}(2\hat{\mathbf{i}}+\hat{\mathbf{k}})$$
$$=\frac{-1}{3}(\hat{\mathbf{i}}+6\hat{\mathbf{j}}+2\hat{\mathbf{k}})$$

Question24

Angle between a diagonal of a cube and a diagonal of its face which are coterminus is

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Options:

A.

$\frac{\pi}{2}$

B.

$\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

C.

$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

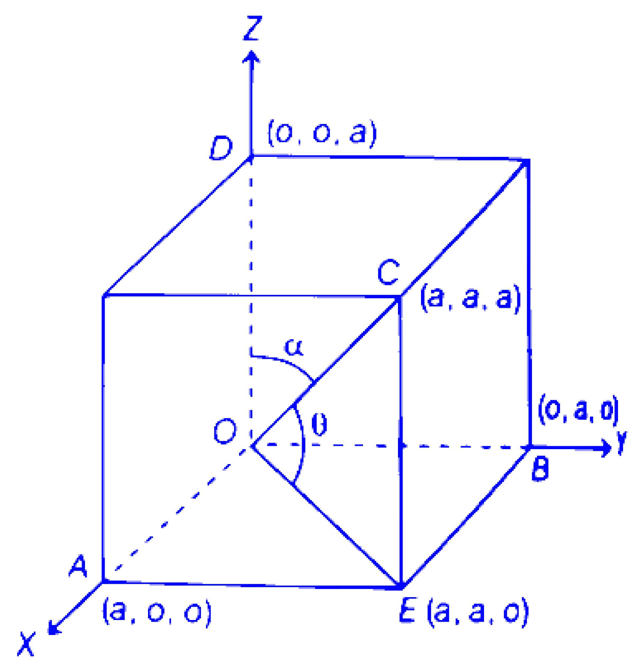
D.

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer: B

Solution:

$$|\mathbf{OD}| = \sqrt{0^2+0^2+a^2} = a$$
$$|\mathbf{OC}| = \sqrt{a^2+a^2+a^2} = a\sqrt{3}$$



$$\Rightarrow \quad \cos\alpha = \frac{a^2}{a\sqrt{3}\cdot a} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$
$$|OE| = a\sqrt{2}$$
$$\cos\theta = \frac{2a^2}{\sqrt{2}\cdot\sqrt{3}a^2} \Rightarrow \theta = \cos^{-1}\sqrt{\frac{2}{3}}$$

Question25

A plane π is passing through the points $A(1,-2,3)$ and $B(6,4,5)$. If the plane π is perpendicular the plane $3x-y+z=2$, then the perpendicular distance from $(0,0,0)$ to the plane π is

Options:

A.

$\frac{63}{\sqrt{594}}$

B.

$\frac{32}{\sqrt{594}}$

C.

$\frac{72}{\sqrt{435}}$

D.

$\frac{23}{\sqrt{135}}$

Answer: A

Solution:

Plane passing through $A(1,-2,3)$ and $B(6,4,5)$

Normal vector of the given plane π

$\mathbf{n}_1 = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Normal vector of plane

$$\mathbf{n} = \mathbf{AB} \times \mathbf{n} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 5 & 6 & 2 \\ 3 & -1 & 1 \end{vmatrix}$$
$$= 8\hat{\mathbf{i}} + \hat{\mathbf{j}} - 23\hat{\mathbf{k}}$$

Equation of plane π

$$8(x-1) + 1(y+2) - 23(z-3) = 0$$
$$\Rightarrow 8x + y - 23z + 63 = 0$$

Perpendicular distances from $(0,0,0)$ to the plane π

$$d = \frac{63}{\sqrt{64+1+529}} = \frac{63}{\sqrt{594}}$$

Question26

The point of intersection of the lines represented by $\mathbf{r}=(\hat{\mathbf{i}}-6\hat{\mathbf{j}}+2\hat{\mathbf{k}})+\mathbf{t}(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})$ and $\mathbf{r}=(4\hat{\mathbf{j}}+\hat{\mathbf{k}})+\mathbf{s}(2\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ is

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Options:

A.

$8\hat{\mathbf{i}}+9\hat{\mathbf{j}}+10\hat{\mathbf{k}}$

B.

$8\hat{\mathbf{i}}+8\hat{\mathbf{j}}+7\hat{\mathbf{k}}$

C.

$8\hat{\mathbf{i}}+9\hat{\mathbf{j}}+8\hat{\mathbf{k}}$

D.

$8\hat{\mathbf{i}}+8\hat{\mathbf{j}}+9\hat{\mathbf{k}}$

Answer: D

Solution:

Let P and Q be the points on the given lines where they intersect.

$P=(1+t,-6+2t,2+t)$ and
$Q=(2s,4+s,1+2s)$

Now, P and Q are identical

$$\begin{aligned} \therefore \quad t+1&=2s & \ldots(i)\\ -6+2t&=4+s & \ldots(ii)\\ 2+t&=1+2s & \ldots(iii) \end{aligned}$$

On solving Eqs. (i) and (ii), we get

$s=4$ and $t=7$

Which satisfy the Eq. (iii)

$\therefore$ Point of intersection $(8, 8, 9)$

$= 8\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

Question27

If the four points $(6, 2, 4), (1, 3, 5), (1, -2, 3)$ and $(6, k, 2)$ are coplanar, then $k =$

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Options:

A.

-5

B.

4

C.

-3

D.

1

Answer: C

Solution:

Let four points $A(6, 2, 4), B(1, 3, 5), C(1, -2, 3)$ and $D(6, k, 2)$

$$\mathbf{AB} = -5\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$
$$\mathbf{AC} = -5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$$
$$\mathbf{AD} = (k - 2)\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

Given, four points ABC and D are coplanar

$\therefore [\mathbf{AB}, \mathbf{AC}, \mathbf{AD}] = 0$

$$\Rightarrow \begin{vmatrix} -5 & 1 & 1 \\ -5 & -4 & -1 \\ 0 & k-2 & -2 \end{vmatrix} = 0$$

$\Rightarrow -5(8 + k - 2) - 1(10) + 1(-5k + 10) = 0$

$\Rightarrow -5(6 + k) - 10 - 5k + 10 = 0$

$\Rightarrow -30 - 5k - 10 - 5k + 10 = 0$

$\Rightarrow -10k - 30 = 0$

$\therefore k = -3$

Question28

$G(1, 0, 1)$ is the centroid of the $\triangle ABC$. If $A = (1, -4, 2)$ and $B = (3, 1, 0)$, then $AG^2 + CG^2 =$

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Options:

A.

BG^2

B.

$2BG^2$

C.

$6BG^2$

D.

$5BG^2$

Answer: D

Solution:

Given, $G = (1, 0, 1)$, $A(1, -4, 2)$ and

$B = (3, 1, 0)$,

then $C = (-1, 3, 1)$

$$AG^2 = (17) \text{ and } CG^2 = 13$$
$$AG^2 + CG^2 = 17 + 13 = 30$$
$$BG^2 = 6$$
$$\because AG^2 + CG^2 = 5 \cdot BG^2$$

Question29

If the sum of the distances of the point $(3, 4, \alpha), \alpha \in R$ from X-axis, Y-axis and Z-axis is minimum, then $\sec \alpha =$

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Options:

A.

2

B.

1

C.

0

D.

-1

Answer: B

Solution:

We have, $(3, 4, \infty)$

Distance of point to X-axis $= \sqrt{16 + \alpha^2}$

Distance of point to Y-axis $= \sqrt{9 + \alpha^2}$

Distance of point to Z-axis $= \sqrt{9 + 16} = 5$

Sum $= S = \sqrt{16 + \alpha^2} + \sqrt{9 + \alpha^2} + 5$

The sum S is minimum when $\alpha = 0$ because $\sqrt{16+\alpha^2}$ and $\sqrt{9+\alpha^2}$ are minimum when $\alpha = 0$

$\because \quad \sec 0 = 1$

Question30

If the equation of the plane passing through the point $(2, -1, 3)$ and perpendicular to each of the planes $3x - 2y + z = 8$ and $x + y + z = 6$ is $lx + my + nz = 1$, then $4m + 2n - 3l =$

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Options:

A.

0

B.

$\frac{-20}{11}$

C.

1

D.

3

Answer: C

Solution:

Let equation of required plane is

$a(x-2) + b(y+1) + c(z-3) = 0$

Now, the plane is perpendicular to

$3x - 2y + z = 8$ and $x + y + z = 6$

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \hat{\mathbf{i}}(-3) - \hat{\mathbf{j}}(2) + k(5)$$

$\because a, b$ and c are $-3, -2$ and 5

$$\begin{aligned}
& -3(x-2)-2(y+1)+5(z-3)=0 \\
\Rightarrow & -3x-2y+5z+6-2-15=0 \\
\Rightarrow & -3x-2y+5z-11=0 \\
\Rightarrow & -3x-2y+5z=11 \\
\Rightarrow & -\frac{3}{11}x-\frac{2}{11}y+\frac{5}{11}z=1 \\
& l=-\frac{3}{11}, m=-\frac{2}{11}, n=\frac{5}{11} \\
& \therefore 4m+2n-3l=-\frac{8}{11}+\frac{10}{11}+\frac{9}{11}=\frac{11}{11}=1
\end{aligned}$$

Question31

The length of the internal bisector of angle A in $\triangle ABC$ with vertices $A(4,7,8), B(2,3,4)$ and $C(2,5,7)$ is

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Options:

A. $\frac{1}{3}\sqrt{29}$

B. $\frac{2}{3}\sqrt{29}$

C. $\frac{2}{3}\sqrt{34}$

D. $\frac{4}{3}\sqrt{34}$

Answer: C

Solution:

Given the points $A(4,7,8)$, $B(2,3,4)$, and $C(2,5,7)$, we need to find the length of the internal bisector of angle A in $\triangle ABC$.

Let's denote the intersection of the angle bisector at A with the line segment BC as point D. The angle bisector divides BC in the ratio $|AB| : |AC|$.

First, calculate the vectors $\mathbf{AB}$ and $\mathbf{AC}$:

$$\mathbf{AB}=B-A=(2-4)\mathbf{i}+(3-7)\mathbf{j}+(4-8)\mathbf{k}=-2\mathbf{i}-4\mathbf{j}-4\mathbf{k},$$
$$\mathbf{AC}=C-A=(2-4)\mathbf{i}+(5-7)\mathbf{j}+(7-8)\mathbf{k}=-2\mathbf{i}-2\mathbf{j}-\mathbf{k}.$$

Now, calculate the magnitudes $|\mathbf{AB}|$ and $|\mathbf{AC}|$:

$$|\mathbf{AB}| = \sqrt{(-2)^2 + (-4)^2 + (-4)^2} = \sqrt{4 + 16 + 16} = 6,$$
$$|\mathbf{AC}| = \sqrt{(-2)^2 + (-2)^2 + (-1)^2} = \sqrt{4 + 4 + 1} = 3.$$

Using these magnitudes, we find the position vector of D:

$$\begin{aligned}\text{Position vector of } D &= \frac{|\mathbf{AC}| \cdot \text{PV of } B + |\mathbf{AB}| \cdot \text{PV of } C}{|\mathbf{AB}| + |\mathbf{AC}|} \\ &= \frac{3(2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + 6(2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k})}{9} \\ &= \frac{6\mathbf{i} + 9\mathbf{j} + 12\mathbf{k} + 12\mathbf{i} + 30\mathbf{j} + 42\mathbf{k}}{9} \\ &= \frac{18\mathbf{i} + 39\mathbf{j} + 54\mathbf{k}}{9} \\ &= 2\mathbf{i} + \frac{13}{3}\mathbf{j} + 6\mathbf{k}.\end{aligned}$$

The vector $\mathbf{AD}$ is:

$$\mathbf{AD} = D - A = (2-4)\mathbf{i} + \left(\tfrac{13}{3} - 7\right)\mathbf{j} + (6-8)\mathbf{k} = -2\mathbf{i} - \tfrac{8}{3}\mathbf{j} - 2\mathbf{k}.$$

Calculate the length $|\mathbf{AD}|$:

$$\begin{aligned}|\mathbf{AD}| &= \sqrt{(-2)^2 + \left(-\frac{8}{3}\right)^2 + (-2)^2} \\ &= \sqrt{4 + \frac{64}{9} + 4} \\ &= \sqrt{\frac{36}{9} + \frac{64}{9} + \frac{36}{9}} \\ &= \sqrt{\frac{136}{9}} \\ &= \frac{\sqrt{136}}{3} \\ &= \frac{2\sqrt{34}}{3} \text{ units.}\end{aligned}$$

Question32

If the direction cosines of lines are given by $l + m + n = 0$ and $mn - 2lm - 2nl = 0$, then the acute angle between those lines is

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Options:

A. $2\pi/5$

B. $\pi/3$

C. $\pi/4$

D. $\pi/60$

Answer: B

Solution:

We have, $l+m+n=0$

and $mn-2lm-2nl=0$

$$\begin{aligned}
\Rightarrow & \quad mn-21(m+n)=0 \\
\Rightarrow & \quad mn+2(m+n)(m+n)=0 \\
\Rightarrow & \quad 2\left(m^2+n^2+2mn\right)+mn=0 \\
\Rightarrow & \quad 2m^2+2n^2+5mn=0 \\
\Rightarrow & \quad 2m^2+5mn+2n^2=0 \\
\Rightarrow & \quad 2m^2+4mn+mn+2n^2=0 \\
\Rightarrow & \quad 2m(m+2n)+n(m+2n)=0 \\
\Rightarrow & \quad (m+2n)(2m+n)=0 \\
\Rightarrow & \quad m+2n=0 \text{ or } 2m+n=0
\end{aligned}$$

When, $m+2n=0 \Rightarrow m=-2n$

Then, $l+(-2n)+n=0 \Rightarrow l=n$

$\Rightarrow \quad \frac{l}{1}=\frac{m}{-2}=\frac{n}{1}$

When, $2m+n=0 \Rightarrow 2m=-n$

Then, $l+m+(-2m)=0$

$$\begin{aligned}
\Rightarrow & \quad l-m=0 \Rightarrow l=m \\
\Rightarrow & \quad \frac{l}{1}=\frac{m}{1}=\frac{n}{-2} \\
\therefore \cos\theta & =\frac{|1-2-2|}{\sqrt{1+4+1}\sqrt{1+1+4}}=\frac{1}{2} \\
\Rightarrow & \quad \theta=\frac{\pi}{3}
\end{aligned}$$

Question33

If the angle θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta=\frac{1}{3'}$ then the value of $\lambda=$

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Options:

A. $\frac{3}{5}$

B. $\frac{5}{4}$

C. $\frac{5}{3}$

D. $\frac{4}{3}$

Answer: C

Solution:

Given, equation of lines

$$\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$$

and plane $2x-y+\sqrt{\lambda}z+4=0$

$$\therefore \sin\theta=\frac{1\times 2+2\times(-1)+2\times\sqrt{\lambda}}{\sqrt{1+4+4}\sqrt{4+1+\lambda}}$$

$$\Rightarrow \quad \frac{1}{3}=\frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$$

$$\Rightarrow \quad \frac{1}{3}=\frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}$$

$$\Rightarrow \quad \sqrt{5+\lambda}=2\sqrt{\lambda}$$

$$\Rightarrow \quad 5+\lambda=4\lambda$$

$$\Rightarrow \quad \lambda=\frac{5}{3}$$

Question34

If $A=(1,2,3)$, $B=(3,4,7)$ and $C=(-3,-2,-5)$ are three points, then the ratio in which the point C divides AB externally is

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Options:

A. 2 : 3

B. 3 : 2

C. 4 : 3

D. 3 : 4

Answer: A

Solution:

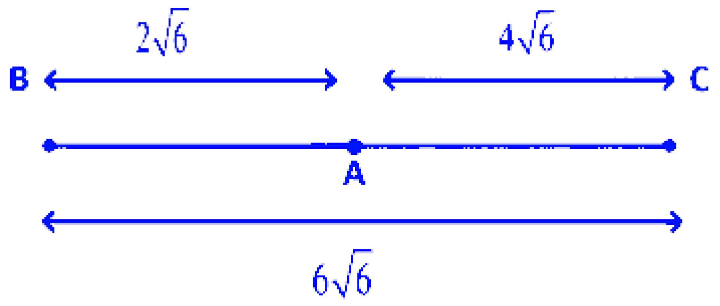
Given, $A(1,2,3), B(3,4,7),$
$C(-3,-2,-5)$
$\Rightarrow \quad \mathbf{AB} = \mathbf{OB} - \mathbf{OA}$
$= 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$
$\mathbf{BC} = \mathbf{OC} - \mathbf{OB}$
$= -6\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 12\hat{\mathbf{k}}$

Let A, B and C be in a line

Then,

$|\mathbf{AB} \cdot \mathbf{BC}| = |\mathbf{AB}| \times |\mathbf{BC}|$
So, $|\mathbf{AB} \cdot \mathbf{BC}| = |-12 - 12 - 48| = +72$
Now, $|\mathbf{AB}| = \sqrt{4+4+16} = \sqrt{24}$
$|\mathbf{BC}| = \sqrt{36+36+144} = \sqrt{216}$
$|\mathbf{AB}| \times |\mathbf{BC}| = \sqrt{24} \times \sqrt{216} = 72$

So, ABC are collinear.



So, C divide AB in ratio 2:3 externally.

Question35

If $\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}, \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $2\hat{\mathbf{i}} + \hat{\mathbf{j}}$ are the vertices of a tetrahedron, then its volume is

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Options:

A. $1/6$

B. $2/3$

C. 3

D. $1/3$

Answer: D

Solution:

Given that

$P(\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}}), Q(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}), R(\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

and $S(2\hat{\mathbf{i}}+\hat{\mathbf{j}})$ are vertices of a tetrahedron

volume $=\frac{1}{6}[\mathbf{PQ} \quad \mathbf{PR} \quad \mathbf{PS}]$

$$=\frac{1}{6}[(2\hat{\mathbf{j}}+2\hat{\mathbf{k}}) \quad (2\hat{\mathbf{j}}+3\hat{\mathbf{k}}) \quad (\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}})]$$

$$=\frac{1}{6}\begin{vmatrix} 0 & 2 & 2 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix}=\frac{1}{6}[-2(0-3)+2(0-2)]$$

$$=\frac{1}{6}(6-4)=\frac{2}{6}=\frac{1}{3}$$

Question36

If a line L makes angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with Y-axis and Z-axis respectively, then the angle between L and another line having direction ratio $1, 1, 1$ is

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Options:

A. $\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$

B. $\cos^{-1}\left(\frac{\sqrt{2}+1}{3\sqrt{3}}\right)$

C. $\cos^{-1}\left(\frac{\sqrt{2}-1}{3}\right)$

D. $\cos^{-1}\left(\frac{\sqrt{2}+1}{\sqrt{6}}\right)$

Answer: D

Solution:

Let the direction cosine of line be
(l, m, n)
Then, $l = \cos\alpha$
$m = \cos\beta$
and $n = \cos\gamma$

Given, $\beta = \frac{\pi}{3}, \gamma = \frac{\pi}{4}$
$\Rightarrow m = \cos\frac{\pi}{3} = \frac{1}{2}$
and $n = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$
$\therefore l^2 + m^2 + n^2 = 1$
$\Rightarrow l^2 + \frac{1}{4} + \frac{1}{2} = 1 \Rightarrow l^2 = 1 - \frac{3}{4} = \frac{1}{4}$
$l = \pm\frac{1}{2}$
$\Rightarrow (l, m, n) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

Direction ratio of second line is $(1, 1, 1)$ So, direction cosine

$(l', m', n') = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Now, angle between two line

$$\cos\theta = l' + mm' + nn'$$
$$= \frac{1}{2\sqrt{3}} + \frac{1}{2\sqrt{3}} + \frac{1}{\sqrt{6}}$$

$$= \frac{1+1+\sqrt{2}}{2\sqrt{3}} = \frac{2+\sqrt{2}}{2\sqrt{3}} = \frac{1+\sqrt{2}}{\sqrt{6}}$$
$$\theta = \cos^{-1}\left(\frac{1+\sqrt{2}}{\sqrt{6}}\right)$$

Question37

If l, m and n are the direction cosines of a line that is perpendicular to the lines having the direction ratios $(1, 2, -1)$ and $(1, -2, 1)$, then $(l + m + n)^2$ is equal to

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Options:

A. $\frac{1}{20}$

B. $\frac{9}{5}$

C. $\frac{1}{5}$

D. $\frac{3}{20}$

Answer: B

Solution:

Given:

A line with direction ratios $(1, 2, -1)$ has the direction cosines:

$\left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

Another line with direction ratios $(1, -2, 1)$ has the direction cosines:

$\left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

A line with direction cosines (l, m, n) is perpendicular to both given lines. This gives us the following conditions based on dot product orthogonality:

$l + 2m - n = 0,$
$l - 2m + n = 0.$

Solving these equations simultaneously, we find:

From the first equation: $l + 2m - n = 0$

From the second equation: $l - 2m + n = 0$

By adding the two equations, we get:

$l + 2m - n + l - 2m + n = 0 \implies 2l = 0 \implies l = 0$

Substitute $l = 0$ back into either equation, say $l + 2m - n = 0$:

$0 + 2m - n = 0 \implies n = 2m$

Next, using the condition for direction cosines:

$l^2 + m^2 + n^2 = 1$

Substitute $l = 0$ and $n = 2m$:

$0^2 + m^2 + (2m)^2 = 1 \implies m^2 + 4m^2 = 1 \implies 5m^2 = 1 \implies m = \pm\frac{1}{\sqrt{5}}$

Also, with $m = \pm\frac{1}{\sqrt{5}}$, it follows that:

$n = \frac{2}{\sqrt{5}}$

Finally, calculate $(l + m + n)^2$:

$(l + m + n)^2 = \left(0 + \frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}}\right)^2 = \left(\frac{3}{\sqrt{5}}\right)^2 = \frac{9}{5}$

Thus, $(l + m + n)^2 = \frac{9}{5}$.

Question38

The foot of the perpendicular drawn from a point $A(1, 1, 1)$ on to a plane π is $P(-3, 3, 5)$.If the equation of the plane parallel to the plane of π and passing through the mid-point of AP is $ax - y + cz + d = 0$, then $a + c - d$ is equal to

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Options:

A. -10

B. 5

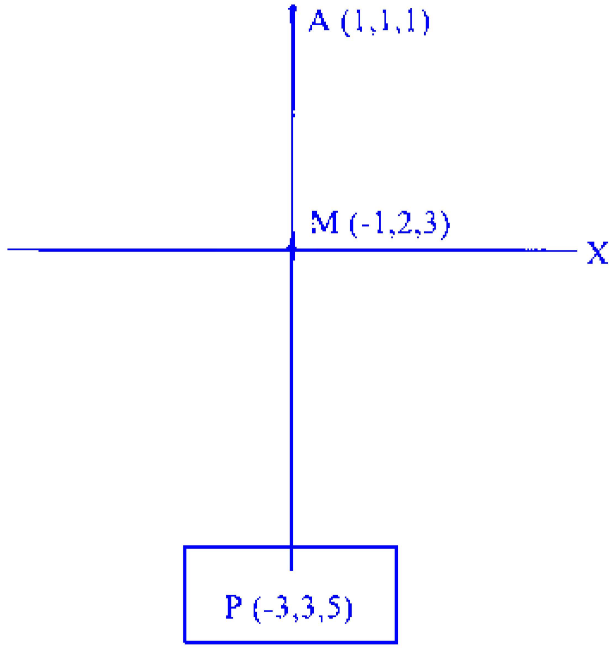
C. -12

D. 2

Answer: A

Solution:

Given, $A(1, 1, 1)$ and $P(-3, 3, 5)$



$$Mid - point of AP = \left(\frac{1-3}{2}, \frac{1+3}{2}, \frac{1+5}{2}\right)$$
$$= (-1, 2, 3)$$

and normal vector of $\overrightarrow{\mathbf{AP}} = (-4, 2, 4)$

The plane passing through P with normal vector, then,

$$\begin{aligned} & -4(x-x_1)+2(y-y_1)+4(z-z_1)=0 \\ \Rightarrow \quad & -4x_0+2y+4z=-4x_1+2y_1+4z_1 \\ \Rightarrow \quad & -4x+2y+4z=16+4+20=38 \\ \Rightarrow \quad & -4x+2y+4z=38 \\ \Rightarrow \quad & -2x+y+2z=19 \end{aligned}$$

Since, a plane parallel to $-2x + y + 2z - 19 = 0$ has same normal vector.

Thus, equation of plane passing through $(-1, 2, 3)$

$$2(x+1) - 1(y-2) - 2(z-3) = 0$$
$$\Rightarrow 2x - y - 2z + 10 = 0$$

On comparing with the general form

$$ax - y + cz + d = 0$$
$$\therefore \quad a = 2, c = -2, d = 10$$

Thus, $a + c - d = 2 - 2 - 10 = -10$

Question39

The distance of a point $(2, 3, -5)$ from the plane $\hat{\mathbf{r}} \cdot (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 4$ is

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Options:

A. $\frac{11}{2}$

B. $\frac{11}{\sqrt{29}}$

C. $\frac{15}{\sqrt{29}}$

D. $\frac{11}{\sqrt{38}}$

Answer: C

Solution:

To find the distance of the point $(2, 3, -5)$ from the plane given by $\hat{\mathbf{r}} \cdot (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 4$, we first express the plane equation in its cartesian form:

$4x - 3y + 2z - 4 = 0$

Given the formula for the distance D from a point (x_1, y_1, z_1) to a plane $ax + by + cz + d = 0$:

$D = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

We substitute the values from the point $(2, 3, -5)$ and the plane equation:

$a = 4$, $b = -3$, $c = 2$, $d = -4$

$x_1 = 2$, $y_1 = 3$, $z_1 = -5$

Plug these into the formula:

$D = \frac{|4\times2+(-3)\times3+2\times(-5)-4|}{\sqrt{4^2+(-3)^2+2^2}}$

Simplify the numerator:

$= \frac{|8-9-10-4|}{\sqrt{16+9+4}}$

$= \frac{|-15|}{\sqrt{29}}$

Which yields:

$D = \frac{15}{\sqrt{29}}$ units

Question40

The orthocentre of triangle fromed by points $(2,1,5)$ $(3,2,3)$ and $(4,0,4)$ is

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Options:

A. $(3,1,2)$

B. $(3,2,3)$

C. $(3,1,4)$

D. $(1,4,0)$

Answer: C

Solution:

The given points are $A(2,1,5)$, $B(3,2,3)$, and $C(4,0,4)$.

To find the orthocenter of the triangle formed by these points, we first calculate the length of each side:

Calculating AB:

$$\begin{aligned} AB &= \sqrt{(3-2)^2+(2-1)^2+(3-5)^2} \\ &= \sqrt{1^2+1^2+(-2)^2} \\ &= \sqrt{1+1+4} \\ &= \sqrt{6} \end{aligned}$$

Calculating BC:

$$\begin{aligned} BC &= \sqrt{(4-3)^2+(0-2)^2+(4-3)^2} \\ &= \sqrt{1^2+(-2)^2+1^2} \\ &= \sqrt{1+4+1} \\ &= \sqrt{6} \end{aligned}$$

Calculating AC:

$$
\begin{aligned}
AC &= \sqrt{(4-2)^2 + (0-1)^2 + (4-5)^2} \\
&= \sqrt{2^2 + (-1)^2 + (-1)^2} \\
&= \sqrt{4+1+1} \\
&= \sqrt{6}
\end{aligned}
$$

Since all sides are equal, the triangle is equilateral. For an equilateral triangle, the orthocenter coincides with the centroid.

The centroid P of the triangle is given by:

$$
\begin{aligned}
P &= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right) \\
&= \left(\frac{2+3+4}{3}, \frac{1+2+0}{3}, \frac{5+3+4}{3}\right) \\
&= \left(\frac{9}{3}, \frac{3}{3}, \frac{12}{3}\right) \\
&= (3, 1, 4)
\end{aligned}
$$

Therefore, the orthocenter of the triangle is $(3, 1, 4)$.

Question41

If $P = (0, 1, 2)$, $Q = (4, -2, 1)$, and $O = (0, 0, 0)$, then $\angle POQ =$

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Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

Solution:

Given the points $P = (0, 1, 2)$, $Q = (4, -2, 1)$, and $O = (0, 0, 0)$, we need to determine the angle $\angle POQ$.

First, calculate the vectors $\mathbf{OP}$ and $\mathbf{OQ}$:

$$\mathbf{OP} = 0\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

$\mathbf{OQ} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

The dot product of the vectors $\mathbf{OP}$ and $\mathbf{OQ}$ is given by:

$\mathbf{OP} \cdot \mathbf{OQ} = (0 \cdot 4) + (1 \cdot -2) + (2 \cdot 1) = -2 + 2 = 0$

The magnitudes of $\mathbf{OP}$ and $\mathbf{OQ}$ are calculated as follows:

$|\mathbf{OP}| = \sqrt{0^2 + 1^2 + 2^2} = \sqrt{5}$

$|\mathbf{OQ}| = \sqrt{4^2 + (-2)^2 + 1^2} = \sqrt{21}$

The angle θ between $\mathbf{OP}$ and $\mathbf{OQ}$ is found using the dot product formula:

$\mathbf{OP} \cdot \mathbf{OQ} = |\mathbf{OP}| \, |\mathbf{OQ}| \cos\theta$

Substituting known values, we have:

$0 = \sqrt{5}\sqrt{21}\cos\theta$

Since the dot product is zero, $\cos\theta = 0$. This implies:

$\theta = \frac{\pi}{2}$

Thus, the angle $\angle POQ$ is $\frac{\pi}{2}$.

Question42

If the perpendicular distance from $(1, 2, 4)$ to the plane $2x + 2y - z + k = 0$ is 3 , then $k =$

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Options:

A. 4

B. 7

C. 9

D. 19

Answer: B

Solution:

To find the value of k, we start with the equation of the plane:

$2x + 2y - z + k = 0$

We know the perpendicular distance D from the point $P(1, 2, 4)$ to the plane is 3. The formula for the perpendicular distance from a point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is given by:

$$D = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

Substitute the values from our plane and point into the formula. Here, $a = 2$, $b = 2$, $c = -1$, and $d = k$. Also, $x_1 = 1$, $y_1 = 2$, and $z_1 = 4$.

$$3 = \left| \frac{2(1) + 2(2) + (-1)(4) + k}{\sqrt{2^2 + 2^2 + (-1)^2}} \right|$$

Calculate inside the absolute value:

$$2 \times 1 + 2 \times 2 + (-1) \times 4 + k = 2 + 4 - 4 + k = 2 + k$$

Calculate the denominator:

$$\sqrt{2^2 + 2^2 + (-1)^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$$

The distance formula simplifies to:

$$3 = \left| \frac{2 + k}{3} \right|$$

Multiply both sides by 3 to eliminate the fraction:

$$9 = |2 + k|$$

This gives two possible equations:

$$2 + k = 9$$

$$2 + k = -9$$

Solve each equation for k:

$$k = 9 - 2 = 7$$

$$k = -9 - 2 = -11$$

Since we're considering the positive solution that makes physical sense in most geometric contexts, we take $k = 7$.

Question43

Angle between the planes, $\mathbf{r} \cdot (12\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 5$ and, $\mathbf{r} \cdot (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = 7$ is

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Options:

A. $\cos^{-1}\left(\frac{12}{13}\right)$

B. $\cos^{-1}\left(\frac{6\sqrt{2}}{13}\right)$

C. $\cos^{-1}\left(\frac{3\sqrt{2}}{13}\right)$

D. $\cos^{-1}\left(\frac{6}{13}\right)$

Answer: B

Solution:

We have, planes

$$P_1 = \mathbf{r} \cdot (12\hat{i} + 4\hat{j} - 3\hat{k}) = 5 \text{ and} \qquad \ldots(i)$$
$$P_2 = \mathbf{r} \cdot (5\hat{i} + 3\hat{j} - 4\hat{k}) = 7 \qquad \ldots(ii)$$

From planes (i) and (ii), we get

$a_1 = 12, b_1 = 4, c_1 = -3 \Rightarrow a_2 = 5, b_2 = 3, c_2 = 4$

We know that

$$\Rightarrow \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$
$$\Rightarrow \cos\theta = \frac{12 \times 5 + 4 \times 3 - 3 \times 3}{\sqrt{144 + 16 + 12}\sqrt{25 + 9 + 16}}$$
$$\Rightarrow \cos\theta = \frac{60}{\sqrt{169}\sqrt{50}} = \frac{60}{13 \times 5\sqrt{2}} \Rightarrow \cos\theta = \frac{6\sqrt{2}}{13} \Rightarrow \theta = \cos^{-1}\left(\frac{6\sqrt{2}}{13}\right)$$

Angle between the planes is $\theta = \cos^{-1}\left(\frac{6\sqrt{2}}{13}\right)$

Question44

The shortest distance between the skew lines $\mathbf{r} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}}) + t(\hat{\mathbf{i}} + 2\hat{\mathbf{k}})$ and $\mathbf{r} = (-2\hat{\mathbf{i}} + \hat{\mathbf{k}}) + s(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$ is

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Options:

A. $\frac{3\sqrt{2}}{\sqrt{7}}$

B. $\frac{3}{\sqrt{7}}$

C. $\frac{3}{\sqrt{14}}$

D. $\frac{4}{\sqrt{14}}$

Answer: A

Solution:

Given skew line

$\mathbf{r}=(2\hat{\mathbf{i}}-\hat{\mathbf{j}})+t(\hat{\mathbf{i}}+2\hat{\mathbf{k}}) \Rightarrow \mathbf{r}=(-2\hat{\mathbf{i}}+\hat{\mathbf{k}})+s(\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}})$

We know that shortest distance between lines vector equations. $\mathbf{r}=\mathbf{a}_1+t\mathbf{b}_1\ldots$ and $\mathbf{r}=\mathbf{a}_2+s\mathbf{b}_2\ldots$ is

$\left|\frac{(\mathbf{b}_1\times\mathbf{b}_2)\cdot(\mathbf{a}_2-\mathbf{a}_1)}{(\mathbf{b}_1\times\mathbf{b}_2)}\right|$

Now, $\mathbf{r}=(2\hat{\mathbf{i}}-\hat{\mathbf{j}})+t(\hat{\mathbf{i}}+2K)$ comparing with

$\mathbf{r}=\mathbf{a}_1+t\mathbf{b}_1 \Rightarrow \mathbf{a}_1=2\hat{\mathbf{i}}-\hat{\mathbf{j}}, \mathbf{b}_1=\hat{\mathbf{i}}+2\hat{\mathbf{k}}$

and $\mathbf{r}=(-2\hat{\mathbf{i}}+\hat{\mathbf{k}})+s(\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}})$ comparing with

$$\begin{aligned} &\mathbf{r}=\mathbf{a}_2+t\mathbf{b}_2 \\ \therefore\quad &\mathbf{a}_2=-2\hat{\mathbf{i}}+\hat{\mathbf{k}}, \mathbf{b}_2=\hat{\mathbf{i}}-\hat{\mathbf{j}}-\hat{\mathbf{k}} \end{aligned}$$

Now, $(\mathbf{a}_2-\mathbf{a}_1)=(-2\hat{\mathbf{i}}+\hat{\mathbf{k}}-2\hat{\mathbf{i}}+\hat{\mathbf{j}})=-4\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

$$\mathbf{b}_1\times\mathbf{b}_2=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 0 & 2 \\ 1 & -1 & -1\end{vmatrix}=\hat{\mathbf{i}}(2)-\hat{\mathbf{j}}(-1-2)+\hat{\mathbf{k}}(-1)=2\hat{\mathbf{i}}+3\mathbf{j}\cdot\hat{\mathbf{i}}$$

$$|\mathbf{b}_1\times\mathbf{b}_2|=\sqrt{2^2+3^2+1^2}=\sqrt{4+9+1}=\sqrt{14}$$

$$\begin{aligned} \text{So, shortest distance } &= \left|\frac{(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}})(-4\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})}{\sqrt{14}}\right| \\ &= \left|\frac{-8+3-1}{\sqrt{14}}\right|=\frac{6}{\sqrt{14}}=\frac{3\sqrt{2}}{\sqrt{7}} \end{aligned}$$

Hence, the shortest distance between skew lines is $\dfrac{3\sqrt{2}}{\sqrt{7}}$

Question45

If the plane $x-y+z+4=0$ divides the line joining the points $P(2,3,-1)$ and $Q(1,4,-2)$ in the ratio $l:m$, then $l+m$ is

Options:

A. 1

B. 3

C. -1

D. 4

Answer: B

Solution:

To solve the problem of determining how the plane $x - y + z + 4 = 0$ divides the line segment connecting points $P(2, 3, -1)$ and $Q(1, 4, -2)$, we represent the division in the ratio $l : m$.

The coordinate of the point R that divides the line segment from P to Q in the ratio $l : m$ is given by:

$R\left(\frac{l\cdot1+m\cdot2}{l+m}, \frac{l\cdot4+m\cdot3}{l+m}, \frac{-2l-m}{l+m}\right)$

Since R lies on the plane defined by the equation $x - y + z + 4 = 0$, substituting the coordinates of R into the plane equation yields:

$\left(\frac{l+2m}{l+m}\right) - \left(\frac{4l+3m}{l+m}\right) + \left(\frac{-2l-m}{l+m}\right) + 4 = 0$

Simplifying this, we get:

$\frac{l+2m-4l-3m-2l-m}{l+m} + 4 = 0$

Simplifying further, we obtain:

$\frac{-5l-2m+4l+4m}{l+m} = 0$

Which reduces to:

$-l + 2m = 0$

From this, it follows that:

$l = 2m$

Therefore, the ratio $l : m$ is $2 : 1$, and consequently:

$l + m = 2 + 1 = 3$

Question46

If the line with direction ratios $(1, \alpha, \beta)$ is perpendicular to the line with direction ratios $(-1, 2, 1)$ and parallel to the line with direction ratios $(\alpha, 1, \beta)$ then (α, β) is

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Options:

A. $(-1, -1)$

B. $(1, -1)$

C. $(-1, 3)$

D. $(1, 1)$

Answer: B

Solution:

To solve this problem, let's consider the given lines and their direction ratios:

The direction ratios of line L_1 are $(1, \alpha, \beta)$.

The direction ratios of line L_2 are $(-1, 2, 1)$.

The direction ratios of line L_3 are $(\alpha, 1, \beta)$.

Conditions Given:

Perpendicularity: The line L_1 is perpendicular to line L_2. Two lines are perpendicular if the dot product of their direction ratios equals zero.

$1 \times (-1) + \alpha \times 2 + \beta \times 1 = 0$

This simplifies to:

$-1 + 2\alpha + \beta = 0 \quad \text{(Equation 1)}$

Parallelism: The line L_1 is parallel to line L_3. Two lines are parallel if their direction ratios are proportional.

This implies:

$\frac{1}{\alpha} = \frac{\alpha}{1} = \frac{\beta}{\beta}$

From the proportionality condition:

$\alpha = 1$

Solving Equations:

Using $\alpha = 1$ in Equation 1:

$-1 + 2 \times 1 + \beta = 0$

Simplifying:

$-1 + 2 + \beta = 0 \quad \Rightarrow \quad \beta = -1$

Thus, the values of α and β are:

$(\alpha, \beta) = (1, -1)$

Question47

Let $P(x_1, y_1, z_1)$ be the foot of perpendicular drawn from the point $Q(2, -2, 1)$ to the plane $x - 2y + z = 1$. If d is the perpendicular from the point Q to the plane and $l = x_1 + y_1 + z_1$, then $l + 3d^2$ is

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Options:

A. 5

B. 7

C. 19

D. 26

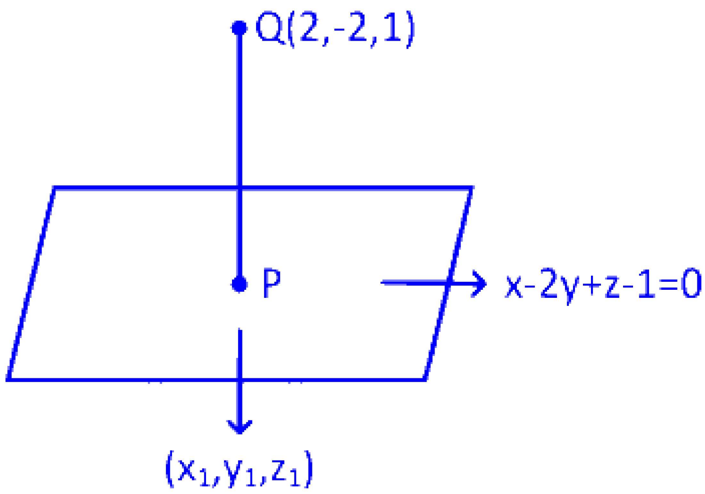
Answer: C

Solution:

We have,

$P(x_1, y_1, z_1)$ be the foot of perpendicular drawn from points $Q(2, -2, 1)$ to the plane

$x - 2y + z = 1$



Equation of PQ is

$$\frac{x_1-2}{1}=\frac{y_1-(-2)}{-2}=\frac{z_1-1}{1}$$
$$\frac{x_1-2}{1}=\frac{y_1+2}{-2}=\frac{z_1-1}{1}=\lambda$$

General points on the line is

$P(\lambda+2-2\lambda-2, \lambda+1) \qquad \ldots(i)$

If P lies on the plane, then it will satisfy the equation

$$\begin{aligned}&\Rightarrow x_1-2y_1+z_1-1=0\\&\Rightarrow(\lambda+2)+2(2\lambda+2)+\lambda+1-1=0\\&\Rightarrow\lambda+2+4\lambda+4+\lambda+1-1=0\\&\Rightarrow 6\lambda+6=0\Rightarrow\lambda=-1\end{aligned}$$

General point on the line is $P(1,0,0)$ If d is the distance between the point P and Q then

$$d=\left|\frac{ax_1+by_1+c_1z_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$
$$\Rightarrow d=\left|\frac{2+4+1-1}{\sqrt{1+4+1}}\right|\Rightarrow d=\left|\frac{6}{\sqrt{6}}\right|$$

Given, $l=x_1+y_1+z_1=1+0+0=1$

Then, $l+3d^2=1+3\left(\frac{6}{\sqrt{6}}\right)^2=1+3\times\frac{36}{6}$

$l+3d^2=19$

Question48

$A(1,2,1), B(2,3,2), C(3,1,3)$ and $D(2,1,3)$ are the vertices of a tetrahedron. If θ is the angle between the faces ABC and ABD, then $\cos\theta=$

Options:

A. $\frac{5}{\sqrt{14}}$

B. $\frac{15}{8\sqrt{7}}$

C. $\frac{3}{\sqrt{14}}$

D. $\frac{5}{2\sqrt{7}}$

Answer: D

Solution:

Vector perpendicular to face ABC.

$$= AB \times AC = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ 2 & -1 & 2 \end{vmatrix}$$
$$= \hat{\mathbf{i}}(2+1) - \hat{\mathbf{j}}(2-2) + \hat{\mathbf{k}}(-1-2)$$
$$= 3\hat{\mathbf{i}} - 0\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

Here, $A(1,2,1), B(2,3,2), C(3,1,3)$ and $D(2,1,3)$

$AB = OB - OA = (1,1,1)$
$AC = (2,-1,2)$
$AD = (1,-1,2)$ and vector perpendicular to face ABD

$$AB \times AD = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$
$$= \hat{\mathbf{i}}(2+1) - \hat{\mathbf{j}}(2-1) + \hat{\mathbf{k}}(-1-1) = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

Since, the angle between the faces = Angle between their normals

Thus, $\cos\theta = \frac{9+0+6}{\sqrt{9+9}\sqrt{9+1+4}}$

$\cos\theta = \dfrac{15}{\sqrt{18}\sqrt{14}}$

$\cos\theta = \dfrac{15}{\sqrt{9\times 2}\sqrt{2\times 7}}$

$\cos\theta = \dfrac{15}{3\sqrt{2}\sqrt{14}} = \dfrac{5}{\sqrt{2}\times\sqrt{2}\times\sqrt{7}}$

$\cos\theta = \dfrac{5}{2\sqrt{7}}$

Question49

Consider the tetrahedron with the vertices $A(3,2,4)$, $B(x_1,y_1,0), C(x_2,y_2,0)$ and $D(x_3,y_3,0)$.If the $\triangle BCD$ is formed by the lines $y=x, x+y=6$ and $y=1$, then the centroid of the tetrahedron is

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Options:

A. $\left(\frac{9}{4},\frac{7}{4},1\right)$

B. $\left(\frac{11}{4},\frac{5}{4},1\right)$

C. $\left(3,\frac{7}{4},1\right)$

D. $(3,2,1)$

Answer: C

Solution:

The tetrahedron is defined by the vertices $A(3,2,4)$, $B(x_1,y_1,0)$, $C(x_2,y_2,0)$, and $D(x_3,y_3,0)$. The triangle $\triangle BCD$ is formed by the lines $y=x$, $x+y=6$, and $y=1$.

To find the centroid of the tetrahedron, we use the formula for the centroid of a tetrahedron with vertices (x_1,y_1,z_1), (x_2,y_2,z_2), (x_3,y_3,z_3), and (x_4,y_4,z_4):

$\left(\frac{x_1+x_2+x_3+x_4}{4},\frac{y_1+y_2+y_3+y_4}{4},\frac{z_1+z_2+z_3+z_4}{4}\right)$

To determine the coordinates of the vertices B, C, and D:

The intersection of the lines $y=x$ and $x+y=6$ gives the point $(3,3)$.

On the line $y=x$, a point such as $(5,1)$ satisfies both conditions line $x+y=6$.

A point on $y = 1$, such as $(1, 1)$, also satisfies the line equation.

Thus, the coordinates of the vertices are:

$(x_2, y_2, z_2) = (3, 3, 0)$

$(x_3, y_3, z_3) = (5, 1, 0)$

$(x_4, y_4, z_4) = (1, 1, 0)$

Substituting these into the centroid formula gives:

$\left(\frac{3+3+5+1}{4}, \frac{2+3+1+1}{4}, \frac{4+0+0+0}{4}\right) = \left(\frac{12}{4}, \frac{7}{4}, \frac{4}{4}\right) = \left(3, \frac{7}{4}, 1\right)$

Therefore, the centroid of the tetrahedron is $\left(3, \frac{7}{4}, 1\right)$.

Question50

If $P(2, \beta, \alpha)$ lies on the plane $x + 2y - z - 2 = 0$ and $Q(\alpha, -1, \beta)$ lies on the plane $2x - y + 3z + 6 = 0$, then the direction cosines of the PQ are

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Options:

A. $\left(-\frac{4}{\sqrt{17}}, 0, \frac{1}{\sqrt{17}}\right)$

B. $\left(+\frac{4}{\sqrt{17}}, 0, \frac{1}{\sqrt{17}}\right)$

C. $\left(\frac{1}{\sqrt{17}}, 0, \frac{4}{\sqrt{17}}\right)$

D. $\left(-\frac{1}{\sqrt{17}}, 0, \frac{4}{\sqrt{17}}\right)$

Answer: A

Solution:

Given, If $P(2, \beta, \alpha)$ lies on the plane $x + 2y - z - 2 = 0$ and $Q(\alpha, -1, \beta)$ lies on the plane $2x - y + 3z + 6 = 0$ direction cosine of line $PQ = ?$

Now, substitute the coordinates of P into the plane equation $(x + 2y - z - 2 = 0)$

$2 + 2\beta - \alpha - 2 = 0 \Rightarrow \alpha = 2\beta$

So, coordinate of P are $(2, \beta, 2\beta)$

Now, substitute the coordinates of $Q(\alpha, -1, \beta)$ into the plane equation

$$\begin{aligned} &(2x - y + 3z + 6 = 0) \\ &\quad 2\alpha + 1 + 3\beta + 6 = 0 \\ &\qquad 2\alpha + 3\beta + 7 = 0 \end{aligned}$$

Coordinate of Q are $(\alpha, -1, \beta)$ direction ratios of the line PQ are given by the differences in the coordinates of Q and P.

$$PQ = (\alpha - 2, -1 - \beta, \beta - 2\beta)$$

Here, $\alpha = -2$ and $\beta = -1$

From Eqs. (i) and (ii), we get

Magnitude of DR's

$$= \sqrt{(\alpha - 2)^2 + (-1 - \beta)^2 + (\beta - 2\beta)^2}$$

Direction cosine of are (l, m, n)

$$l = \frac{\alpha - 2}{\sqrt{(\alpha - 2)^2 + (-1 - \beta)^2 + (\beta - 2\beta)^2}}$$

$$m = \frac{-1 - \beta}{\sqrt{(\alpha - 2)^2 + (-1 - \beta)^2 + (\beta - 2\beta)^2}}$$

$$n = \frac{\beta - 2\beta}{\sqrt{(\alpha - 2)^2 + (-1 - \beta)^2 + (\beta - 2\beta)^2}}$$

$$\text{So, } l = \frac{-2 - 2}{\sqrt{(-2 - 2)^2 + (-1 + 1)^2 + (1)^2}}$$

$$= \frac{-4}{\sqrt{17}}$$

$$m = \frac{-1 + 1}{\sqrt{17}} = 0$$

$$n = \frac{-1 + 2}{\sqrt{17}} = \frac{1}{\sqrt{17}} : \left(\frac{-4}{\sqrt{17}}, 0, \frac{1}{\sqrt{17}}\right)$$

Question51

Let π be the plane that passes through the point $(-2, 1, -1)$ and parallel to the plane $2x - y + 2z = 0$. Then the foot of perpendicular drawn from the point $(1, 2, 1)$ to the plane π is

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Options:

A. $(-3,-1,1)$

B. $(-1,1,-3)$

C. $(-3,3,-1)$

D. $(-1,3,-1)$

Answer: D

Solution:

To find the foot of the perpendicular from the point $(1,2,1)$ to the plane π, which passes through the point $(-2,1,-1)$ and is parallel to the plane $2x-y+2z=0$, we can follow these steps:

Step 1: Determine the Equation of the Plane π

The plane π has the same normal vector as the plane $2x-y+2z=0$, which is $2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$.

So, the general equation of the plane π will be:

$2x-y+2z=d$

Given that the plane passes through the point $(-2,1,-1)$, we substitute these coordinates into the equation:

$2(-2)-1+2(-1)=d$

$-4-1-2=d$

$d=-7$

Thus, the equation of the plane π is:

$2x-y+2z=-7$

Step 2: Find the Foot of the Perpendicular

Let $P(a,b,c)$ be the foot of the perpendicular from the point $Q(1,2,1)$ to the plane. The vector from Q to P, $\overrightarrow{QP}$, must be parallel to the normal vector of the plane, which is given by:

$2\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

Thus, we can express:

$\overrightarrow{QP}=(a-1)\hat{\mathbf{i}}+(b-2)\hat{\mathbf{j}}+(c-1)\hat{\mathbf{k}}$

Equating components, we get the parallel condition:

$\frac{a-1}{2}=\frac{b-2}{-1}=\frac{c-1}{2}=\lambda$

From here, solve for a, b, and c:

$a=2\lambda+1$

$b = 2 - \lambda$

$c = 2\lambda + 1$

Step 3: Use the Plane Equation

Since point $P(a, b, c)$ lies on the plane:

$2(2\lambda + 1) - (2 - \lambda) + 2(2\lambda + 1) = -7$

$4\lambda + 2 - 2 + \lambda + 4\lambda + 2 = -7$

$9\lambda + 2 = -7$

Solving for λ:

$9\lambda = -9$

$\lambda = -1$

Step 4: Substitute to Find Coordinates

Substituting $\lambda = -1$ back into the equations for a, b, and c:

$a = 2(-1) + 1 = -1$

$b = 2 - (-1) = 3$

$c = 2(-1) + 1 = -1$

Thus, the coordinates of the foot of the perpendicular are:

$P(-1, 3, -1)$

Question52

The angle between the line with the direction ratios $(2, 5, 1)$ and the plane $8x + 2y - z = 14$ is

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Options:

A. $\cos^{-1}\left(\frac{64}{\sqrt{9804}}\right)$

B. $\sin^{-1}\left(\frac{64}{\sqrt{9804}}\right)$

C. $\sin^{-1}\left(\frac{25}{\sqrt{2070}}\right)$

D. $\cos^{-1}\left(\frac{25}{\sqrt{2070}}\right)$

Answer: C

Solution:

Given the direction ratios $(2, 5, 1)$ of a line and the equation of the plane $8x + 2y - z = 14$, we aim to find the angle between the line and the plane.

To determine the angle θ between the line and the plane, we use the formula for the sine of the angle:

$$\sin\theta = \frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}}$$

Here, the direction ratios of the line are $(a_1, a_2, a_3) = (2, 5, 1)$ and the normal to the plane, derived from the coefficients of x, y, and z, is $(b_1, b_2, b_3) = (8, 2, -1)$.

Substituting these into the formula, we calculate:

$$\sin\theta = \frac{2\times8+5\times2+1\times(-1)}{\sqrt{2^2+5^2+1^2}\cdot\sqrt{8^2+2^2+(-1)^2}}$$

Simplifying, we get:

$$= \frac{16+10-1}{\sqrt{4+25+1}\cdot\sqrt{64+4+1}}$$

$$= \frac{25}{\sqrt{30}\cdot\sqrt{69}}$$

This simplifies further to:

$$= \frac{25}{3\sqrt{230}} = \frac{25}{\sqrt{2070}}$$

Therefore, the angle θ is given by:

$$\theta = \sin^{-1}\left(\frac{25}{3\sqrt{230}}\right) = \sin^{-1}\left(\frac{25}{\sqrt{2070}}\right)$$

Question53

The direction cosines of the line of intersection of the planes $x + 2y + z - 4 = 0$ and $2x - y + z - 3 = 0$ are

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Options:

A. $\left(\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, \frac{-4}{\sqrt{26}}\right)$

B. $\left(\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$

C. $\left(\frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}, \frac{-5}{\sqrt{35}}\right)$

D. $\left(\frac{3}{\sqrt{22}}, \frac{-2}{\sqrt{22}}, \frac{3}{\sqrt{22}}\right)$

Answer: C

Solution:

To find the direction cosines of the line of intersection of the planes, we start with the given equations:

$x + 2y + z - 4 = 0$

$2x - y + z - 3 = 0$

The direction ratios for these planes are derived from their coefficients, so for equation (i), the direction ratios are $(1, 2, 1)$ and for equation (ii), they are $(2, -1, 1)$.

To find the direction ratios of the line of intersection, we calculate the cross product of these direction ratios using:

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

Calculating the cross product:

$$\begin{aligned} &= (2 \cdot 1 + 1 \cdot 1)\hat{\mathbf{i}} - (1 \cdot 1 - 2 \cdot 1)\hat{\mathbf{j}} + (1 \cdot (-1) - 2 \cdot 2)\hat{\mathbf{k}} \\ &= (2 + 1)\hat{\mathbf{i}} - (1 - 2)\hat{\mathbf{j}} + (-1 - 4)\hat{\mathbf{k}} \\ &= 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 5\hat{\mathbf{k}} \end{aligned}$$

Thus, the direction ratios are $\langle 3, 1, -5 \rangle$.

To convert these into direction cosines, we normalize the vector:

$$\left\langle \frac{3}{\sqrt{9+1+25}}, \frac{1}{\sqrt{9+1+25}}, \frac{-5}{\sqrt{9+1+25}} \right\rangle = \left\langle \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}, \frac{-5}{\sqrt{35}} \right\rangle$$

These are the direction cosines of the line of intersection.

Question54

If L_1 and L_2 are two lines which pass through origin and having direction ratios $(3, 1, -5)$ and $(2, 3, -1)$ respectively, then equation of the plane containing L_1 and L_2 is

Options:

A. $4x + 5y - 63 = 0$

B. $5x - y + 3z = 0$

C. $2x - y + z = 0$

D. $x - 5y + 3z = 0$

Answer: C

Solution:

Given the direction ratios for lines L_1 and L_2 as $(3, 1, -5)$ and $(2, 3, -1)$ respectively, we need to find the equation of the plane that contains both lines L_1 and L_2.

To derive the equation of the plane, we use the determinant method with the direction ratios of the lines as follows:

$$\begin{vmatrix} x & y & z \\ 3 & 1 & -5 \\ 2 & 3 & -1 \end{vmatrix} = 0$$

This expands to:

$x((-1 \times 3) - (15 \times 1)) - y((-3 \times 1) - (10 \times 3)) + z((9 \times 3) - (2 \times (-1)))$

After simplifying, we obtain:

$x(-1 + 15) - y(-3 + 10) + z(9 - 2) = 0$

Which further simplifies to:

$14x - 7y + 7z = 0$

Dividing through by 7, we get the simplified equation of the plane:

$2x - y + z = 0$

Question55

Let $O(\mathbf{O})$, $A(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$, $B(-2\hat{\mathbf{i}} + 3\hat{\mathbf{k}})$, $C(2\hat{\mathbf{i}} + \hat{\mathbf{j}})$ and $D(4\hat{\mathbf{k}})$ are position vectors of the points O, A, B, C and D. If a line passing through A and B intersects the plane passing through O, C and

D at the point R, then position vector of R is

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Options:

A. $-8\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

B. $2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

C. $-7\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

D. $3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

Answer: A

Solution:

To find the position vector of point R, where a line passing through points A and B intersects the plane defined by points O, C, and D, we start by calculating the vector equations and the intersection.

Position vectors of the points are:

$A = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$B = -2\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$

$C = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$D = 4\hat{\mathbf{k}}$

Step 1: Find the vector AB:

$\mathbf{AB} = B - A = (-2\hat{\mathbf{i}} + 3\hat{\mathbf{k}}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) = -3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

Step 2: Parametric equation of the line through A and B:

$\mathbf{r} = \mathbf{A} + t\mathbf{AB} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + t(-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

Simplifying, we get:

$\mathbf{r} = (1 - 3t)\hat{\mathbf{i}} + (2 - 2t)\hat{\mathbf{j}} + (1 + 2t)\hat{\mathbf{k}}$

Step 3: Equation for the plane through O, C, and D:

$\mathbf{OC} = C - O = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$

$\mathbf{OD} = D - O = 4\hat{\mathbf{k}}$

Calculate the normal vector $\mathbf{n}$:

$$\mathbf{n} = \mathbf{OC} \times \mathbf{OD} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 4\hat{\mathbf{i}} - 8\hat{\mathbf{j}}$$

Step 4: Plane equation based on $\mathbf{n}$:

The plane equation is:

$4x - 8y = 0$

Step 5: Substitute the parametric line equation into the plane equation:

$4(1 - 3t) - 8(2 - 2t) = 0$

Simplify the equation:

$4 - 12t - 16 + 16t = 04t - 12 = 0t = 3$

Step 6: Calculate the coordinates of R:

Substitute $t = 3$ into $\mathbf{r} = (1 - 3t)\hat{\mathbf{i}} + (2 - 2t)\hat{\mathbf{j}} + (1 + 2t)\hat{\mathbf{k}}$:

$x = 1 - 3(3) = -8$

$y = 2 - 2(3) = -4$

$z = 1 + 2(3) = 7$

Thus, the position vector of R is:

$-8\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

Question56

The distance of the point $O(\mathbf{O})$ from the plane $\mathbf{r}.\ (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$ measured parallel to $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ is

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Options:

A. 35

B. 30

C. 25

D. 42

Answer: A

Solution:

The distance of the point $O(\mathbf{O})$ from the plane, given by the equation $\mathbf{r} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$, is measured along the direction of the vector $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$.

First, we identify the plane's equation:

$x + y + z = 5$

The line parallel to the vector $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ can be represented parametrically as:

$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6} = K$

This gives us:

$x = 2K, \quad y = 3K, \quad z = -6K$

For the point $(2K, 3K, -6K)$ to be on the plane, it must satisfy the plane equation:

$2K + 3K - 6K = 5$

Solving this, we find:

$-K = 5 \quad \Rightarrow \quad K = -5$

Thus, the coordinates of the point on the line are:

$(-10, -15, 30)$

Now, to find the distance of this point from the origin, we calculate the Euclidean distance:

$$\begin{aligned} \text{Distance} &= \sqrt{(-10)^2 + (-15)^2 + 30^2} \\ &= \sqrt{100 + 225 + 900} \\ &= \sqrt{1225} \\ &= 35 \end{aligned}$$

Therefore, the required distance is 35.

Question57

If $A(1, 0, 2), B(2, 1, 0), C(2, -5, 3)$ and $D(0, 3, 2)$ are four points and the point of intersection of the lines AB and CD is $P(a, b, c)$, then $a + b + c =$

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Options:

A. 3

B. -5

C. 5

D. -3

Answer: A

Solution:

To find the point of intersection $P(a, b, c)$ of the lines AB and CD, let's first determine the equations of these lines.

Equation of Line AB

For line AB, given points $A(1, 0, 2)$ and $B(2, 1, 0)$, the direction ratios are:

$(2-1, 1-0, 0-2) = (1, 1, -2)$

Thus, the equation of line AB is:

$\frac{x-1}{1} = \frac{y-0}{1} = \frac{z-2}{-2}$

Equation of Line CD

For line CD, given points $C(2, -5, 3)$ and $D(0, 3, 2)$, the direction ratios are:

$(0-2, 3+5, 2-3) = (-2, 8, -1)$

Thus, the equation of line CD is:

$\frac{x}{-2} = \frac{y-3}{8} = \frac{z-2}{-1} = K$

Therefore, in parametric form, the point P on line CD can be expressed as:

$(x, y, z) = (2K, 3-8K, K+2)$

Point P on Line AB

Since point P also lies on line AB, we equate the expressions:

$2K - 1 = 3 - 8K = \frac{K}{-2}$

Solving the first two equations:

$2K - 1 = 3 - 8K$

$10K = 4$

$K = \frac{2}{5}$

Substituting $K = \frac{2}{5}$ into the parametric form:

$x = 2K = 2 \times \frac{2}{5} = \frac{4}{5}$

$y = 3 - 8K = 3 - 8 \times \frac{2}{5} = -\frac{1}{5}$

$z = K + 2 = \frac{2}{5} + 2 = \frac{12}{5}$

Thus, the coordinates of P are:

$\left(\frac{4}{5}, -\frac{1}{5}, \frac{12}{5}\right)$

Therefore, the sum $a + b + c$ is:

$a + b + c = \frac{4}{5} - \frac{1}{5} + \frac{12}{5} = 3$

Question58

The direction cosines of two lines are connected by the relations $l + m - n = 0$ and $lm - 2mn + nl = 0$. If θ is the acute angle between those lines, then $\cos\theta =$

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Options:

A. $\frac{\pi}{6}$

B. $\frac{1}{\sqrt{7}}$

C. $\sqrt{\frac{5}{6}}$

D. $\frac{\pi}{3}$

Answer: B

Solution:

We start with the relations for the direction cosines of two lines:

$l + m - n = 0 \quad \text{and} \quad lm - 2mn + nl = 0$

From $l + m - n = 0$, we can express n as:

$n = l + m$

Substituting this into the second equation, we have:

$lm - 2m(l + m) + (l + m)l = 0$

This simplifies to:

$lm - 2lm - 2m^2 + l^2 + lm = 0$

$l^2 = 2m^2$

Thus, $l = \pm\sqrt{2}m$.

Now, we express n based on these values of l:

$n = \pm\sqrt{2}m + m = (\pm\sqrt{2} + 1)m$

We deduce two scenarios for the direction cosines:

$l : m : n = \sqrt{2} : 1 : \sqrt{2} + 1$

$l : m : n = -\sqrt{2} : 1 : -\sqrt{2} + 1$

The cosine of the acute angle θ between the lines is given by:

$\cos\theta = \frac{\sqrt{2}\times(-\sqrt{2})+1\times1+(\sqrt{2}+1)(-\sqrt{2}+1)}{\sqrt{2+1+2+1+2\sqrt{2}}\sqrt{2+1+2+1-2\sqrt{2}}}$

This simplifies as:

$= \frac{|-2+1-1|}{\sqrt{6+2\sqrt{2}}\sqrt{6-2\sqrt{2}}} = \frac{2}{\sqrt{36-8}} = \frac{1}{\sqrt{7}}$

Therefore, the cosine of the acute angle θ is $\frac{1}{\sqrt{7}}$.

Question59

The distance from a point $(1, 1, 1)$ to a variable plane π is 12 units and the points of intersections of the plane π and X, Y, Z -axes are A, B, C respectively, If the point of intersection of the planes through the points A, B, C and parallel to the coordinate planes is P, then the equation of the locus of P is

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Options:

A. $\left(\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}\right) = 143\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)$

B. $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 144$

C. $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} - 1\right)^2 = 144\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)$

D. $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}-1\right)^2=144\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)^2$

Answer: C

Solution:

To determine the locus of point P, consider a plane π intersecting the coordinate axes at points A, B, and C.

The equation of the plane is given by:

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

where the plane intersects the X-axis at $A(a,0,0)$, Y-axis at $B(0,b,0)$ and Z-axis at $C(0,0,c)$.

The distance from a fixed point $(1,1,1)$ to this plane is 12 units. The formula for the distance d from a point (x_1,y_1,z_1) to a plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ is given by:

$d=\frac{\left|\frac{x_1}{a}+\frac{y_1}{b}+\frac{z_1}{c}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$

Substituting the point $(1,1,1)$ into the distance formula, we have:

$\frac{\left|\frac{1}{a}+\frac{1}{b}+\frac{1}{c}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=12$

Squaring both sides, we obtain:

$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}-1\right)^2=144\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)$

Thus, we identify that point P, which is the intersection of planes through points A, B, C and parallel to the coordinate planes, adheres to the following equation:

$\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}-1\right)^2=144\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)$

Therefore, the locus of P satisfies this equation.

Question60

The shortest distance between the skew lines $\mathbf{r}=(-\hat{\mathbf{i}}-2\hat{\mathbf{j}}-3\hat{\mathbf{k}})+t(3\hat{\mathbf{i}}-2\hat{\mathbf{j}}-2\hat{\mathbf{k}})$ and $\mathbf{r}=(7\hat{\mathbf{i}}+4\hat{\mathbf{k}})+s(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+2\hat{\mathbf{k}})$ is

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Options:

A. 15

B. 0

C. 9

D. 16

Answer: C

Solution:

To find the shortest distance between two skew lines given by:

$\mathbf{r} = (-\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + t(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$\mathbf{r} = (7\hat{\mathbf{i}} + 4\hat{\mathbf{k}}) + s(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

follow these steps:

Define the Position Vectors and Direction Vectors:

$\mathbf{a}_1 = -\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\mathbf{b}_1 = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

$\mathbf{a}_2 = 7\hat{\mathbf{i}} + 4\hat{\mathbf{k}}$

$\mathbf{b}_2 = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

Calculate the Vector Connecting Points on the Lines:

$\mathbf{a}_2 - \mathbf{a}_1 = (7+1)\hat{\mathbf{i}} + (0+2)\hat{\mathbf{j}} + (4+3)\hat{\mathbf{k}} = 8\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

Find the Cross Product of the Direction Vectors:

$$\mathbf{b}_1 \times \mathbf{b}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & -2 & -2 \\ 1 & -2 & 2 \end{vmatrix}$$

$= \hat{\mathbf{i}}((-2)\cdot 2 - (-2)\cdot(-2)) - \hat{\mathbf{j}}(3\cdot 2 - (-2)\cdot 1) + \hat{\mathbf{k}}(3\cdot(-2) - (-2)\cdot 1)$

$= \hat{\mathbf{i}}(-4-4) - \hat{\mathbf{j}}(6+2) + \hat{\mathbf{k}}(-6+2)$

$= -8\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

Calculate the Magnitude of the Cross Product:

$|\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{(-8)^2 + (-8)^2 + (-4)^2} = \sqrt{64+64+16} = \sqrt{144} = 12$

Find the Required Distance:

$\text{Distance} = \frac{|(8\hat{\mathbf{i}}+2\hat{\mathbf{j}}+7\hat{\mathbf{k}})\cdot(-8\hat{\mathbf{i}}-8\hat{\mathbf{j}}-4\hat{\mathbf{k}})|}{12}$

$= \frac{|8(-8)+2(-8)+7(-4)|}{12}$

$= \frac{|-64-16-28|}{12}$

$= \frac{108}{12} = 9$

Thus, the shortest distance between these skew lines is 9.

Question61

If $A(1,2,0), B(2,0,1), C(-3,0,2)$ are the vertices of $\triangle ABC$, then the length of the internal bisector of $\angle BAC$ is

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Options:

A. $3\sqrt{6}$

B. $\frac{2\sqrt{14}}{3}$

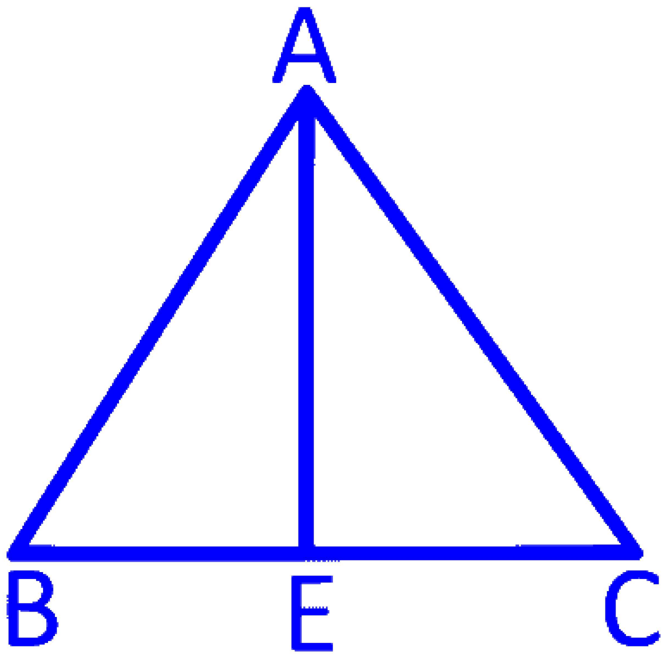
C. $6\sqrt{14}$

D. $\frac{2\sqrt{6}}{3}$

Answer: B

Solution:

Let AE bisects $\angle BAC$.



$$AB = \sqrt{(2-1)^2 + (0-2)^2 + (1-0)^2} = \sqrt{6}$$
$$AC = \sqrt{(-3-1)^2 + (0-2)^2 + (2-0)^2}$$
$$= \sqrt{24} = 2\sqrt{6}$$
$$\frac{BE}{EC} = \frac{AB}{AC} = \frac{\sqrt{6}}{2\sqrt{6}} = \frac{1}{2}$$

$\therefore$ E divides BC in ratio 1:2

By section formula,

Coordinate of E is

$$\left(\frac{-3+4}{1+2}, \frac{0+0}{1+2}, \frac{2+2}{1+2}\right)$$
$\Rightarrow$
$$AB = \sqrt{\left(\left(1-\frac{1}{3}, 0, \frac{4}{3}\right)\right)^2 + (2-0)^2 + \left(0-\frac{4}{3}\right)^2}$$
$$= \sqrt{\frac{4}{9} + 4 + \frac{16}{9}}$$
$$= \sqrt{\frac{56}{9}} = \frac{2\sqrt{14}}{3}$$

Question62

The perpendicular distance from the point $(-1, 1, 0)$ to the line joining the points $(0, 2, 4)$ and $(3, 0, 1)$ is

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Options:

A. 10

B. $\frac{2\sqrt{5}}{5}$

C. $\frac{5}{\sqrt{2}}$

D. 8

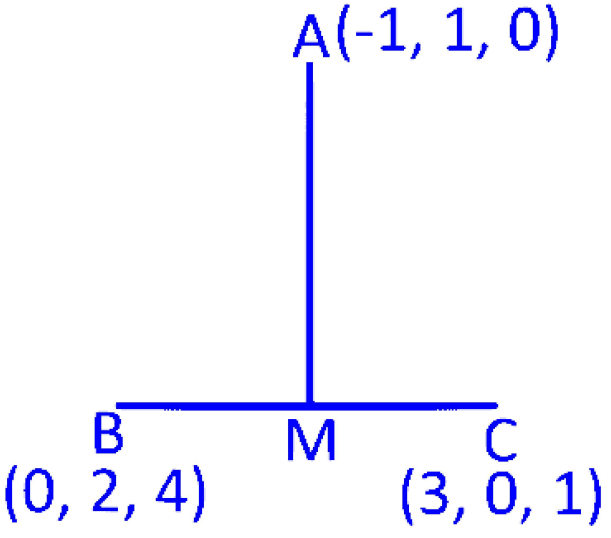
Answer: C

Solution:

Equation of line joining $(0, 2, 4)$ and $(3, 0, 1)$ is

$\frac{x}{3} = \frac{y-2}{-2} = \frac{z-4}{-3} = k$

Let coordinate of foot of perpendicular be $M\ (\ 3k, 2 - 2k, 4 - 3k\ )$



Direction ratio of $BC = \langle 3, -2, -3 \rangle$

Direction ratio of

$AM =< 3k + 1, 1 - 2k, 4 - 3k\rangle$

As AM and BC are perpendicular

$$3(3k+1)-2(1-2k)-3(4-3k)=0$$
$$9k+3-2+4k-12+9k=0$$
$$22k-11=0$$
$$k=\frac{1}{2}$$

$\therefore$ Coordinate of M is $\left(\frac{3}{2}, 1, \frac{5}{2}\right)$

Hence, required length $= AM$

$$=\sqrt{\left(-1-\frac{3}{2}\right)^2+(1-1)^2+\left(0-\frac{5}{2}\right)^2}$$
$$=\sqrt{\frac{25}{4}+\frac{25}{4}}=\sqrt{2}\times\frac{5}{2}=\frac{5}{\sqrt{2}}$$

Question63

A line L passes through the points $(1, 2, -3)$ and $(\beta, 3, 1)$ and a plane π passes through the points $(2, 1, -2)$, $(-2, -3, 6)$, $(0, 2, -1)$. If θ is the angle between the line L and plane π, then $27\cos^2\theta =$

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Options:

A. 25

B. 9

C. 5

D. 2

Answer: D

Solution:

To find the relationship between the line and the plane, first determine their respective equations.

Equation of the Line:

The line L passes through the points $(1, 2, -3)$ and $(\beta, 3, 1)$, but let's assume the correct coordinates at the second point are $(3, 3, -1)$ based on common calculation practices for vector direction. This gives the direction vector as $(3-1, 3-2, -1+3)=(2, 1, 2)$.

The parametric equations of the line can be written as:

$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z+3}{2}$

Equation of the Plane:

The plane π passes through the points $(2, 1, -2)$, $(-2, -3, 6)$, and $(0, 2, -1)$. The normal vector of the plane can be computed using the cross product of two vectors formed by these points:

Vector 1: $(-2-2, -3-1, 6+2) = (-4, -4, 8)$

Vector 2: $(0-2, 2-1, -1+2) = (-2, 1, 1)$

The cross product (normal vector) is:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & -4 & 8 \\ -2 & 1 & 1 \end{vmatrix} = \mathbf{i}((1)(1)-(1)(8)) - \mathbf{j}((-4)(1)-(8)(-2)) + \mathbf{k}((-4)(1)-(-4)(-2))$$

Solving, we find:

$\mathbf{i}(-4) - \mathbf{j}(12) + \mathbf{k}(-12) = (-4, -12, -12)$

Thus, the plane equation is:

$-4(x-2) - 12(y-1) - 12(z+2) = 0$

Expanding and simplifying, we get:

$-4x - 12y - 12z + 8 + 12 + 24 = 0$

$-4x - 12y - 12z + 44 = 0$

It simplifies to:

$x + y + z = 5$

Angle Between Line and Plane:

The direction vector of the line is $(2, 1, 2)$ and the normal vector of the plane is $(1, 1, 1)$.

The angle θ between the line and the plane is given by:

$$\sin\theta = \frac{|2 \cdot 1 + 1 \cdot 1 + 2 \cdot 1|}{\sqrt{2^2+1^2+2^2}\sqrt{1^2+1^2+1^2}} = \frac{|2+1+2|}{\sqrt{9} \cdot \sqrt{3}} = \frac{5}{3\sqrt{3}}$$

Now, compute $\cos^2\theta$:

$27\cos^2\theta = 27(1-\sin^2\theta)$

Substitute $\sin^2\theta$:

$$1 - \sin^2\theta = 1 - \left(\frac{5}{3\sqrt{3}}\right)^2 = 1 - \frac{25}{27} = \frac{2}{27}$$

Therefore,

$27\cos^2\theta = 27 \times \frac{2}{27} = 2$

In conclusion, the value is $\boxed{2}$.

Question64

If the points with position vectors $(\alpha\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 13\hat{\mathbf{k}}), (6\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 11\hat{\mathbf{k}}), \left(\frac{9}{2}\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} - 8\hat{\mathbf{k}}\right)$ are collinear, then $(19\alpha - 6\beta)^2 =$

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Options:

A. 16

B. 36

C. 25

D. 49

Answer: B

Solution:

To determine if the points with the given position vectors are collinear, we need to set up vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ as follows:

Let point A have the position vector $\alpha\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 13\hat{\mathbf{k}}$.

Let point B have the position vector $6\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$.

Let point C have the position vector $\frac{9}{2}\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$.

The vector $\overrightarrow{AB}$ is calculated as:

$$\overrightarrow{AB} = (6 - \alpha)\hat{\mathbf{i}} + (11 - 10)\hat{\mathbf{j}} + (11 - 13)\hat{\mathbf{k}}$$

So:

$$\overrightarrow{AB} = (6 - \alpha)\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

The vector $\overrightarrow{AC}$ is calculated as:

$$\overrightarrow{AC} = (\tfrac{9}{2} - \alpha)\hat{\mathbf{i}} + (\beta - 10)\hat{\mathbf{j}} + (-8 - 13)\hat{\mathbf{k}}$$

So:

$\overrightarrow{AC} = \left(\frac{9}{2} - \alpha\right)\hat{\mathbf{i}} + (\beta - 10)\hat{\mathbf{j}} - 21\hat{\mathbf{k}}$

Since the points are collinear, vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel, which implies:

$\frac{6-\alpha}{\frac{9}{2}-\alpha} = \frac{1}{\beta-10} = \frac{-2}{-21}$

Solving these equations, we find:

$\frac{1}{\beta-10} = \frac{2}{21} \Rightarrow \beta - 10 = \frac{21}{2}$

$\beta = 10 + \frac{21}{2} = \frac{41}{2}$

For the other ratio:

$\frac{12-2\alpha}{9-2\alpha} = \frac{2}{21}$

$126 - 21\alpha = 9 - 2\alpha$

So:

$19\alpha = 117$

$\alpha = \frac{117}{19} = 6$

Given:

$(19\alpha - 6\beta)^2 = (117 - 123)^2 = (-6)^2 = 36$

Thus, the final result is $(19\alpha - 6\beta)^2 = 36$.

Question65

The equation $axy + by = cy$ represents the locus of the points which lie on

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Options:

A. ZX - plane or on the planes perpendicular to $\overline{XX}$ - plene.

B. the planes perpendicular to X-avis.

C. the thes porpendicular to ZX-plene.

D. the lines perpendicular to XX-plane.

Answer: A

Solution:

The given equation is $axy + by = cy$.

We can simplify this to:

$ax + bz = c$

This equation represents a plane. The key characteristic of this plane is its orientation in space.

Since there is no specific term dealing with the y-coordinate in the rearranged equation $ax + bz = c$, this plane is parallel to the Y-axis.

Thus, it is a plane that is perpendicular to the ZX-plane or could coincide with the ZX-plane itself.

In simpler terms, the ZX-plane is defined where $y = 0$, and any plane parallel to it or identical will have no effect from changes in the y-coordinate.

Question66

Let $P(\alpha, 4, 7)$ and $Q(\beta, \beta, 8)$ be two points. If YZ-plane divides the join of the points P and Q in the ratio $2:3$ and ZX-plane divides the join of P and Q in the ratio $4:5$, then length of line segment PQ is

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Options:

A. $\sqrt{107}$

B. $\sqrt{27}$

C. $\sqrt{83}$

D. $\sqrt{97}$

Answer: A

Solution:

We know that $Y\mathbb{Z}$ plane divides PQ in the ratio 2:3.

By section formula the intersecting point is

$\left(\frac{3a+6}{5}, \frac{12+2\beta}{5}, \frac{21+16}{5}\right)$

On Y plane, x-coordinate of any point is zera.

$\Rightarrow 3\alpha + 6 = 0 \Rightarrow \alpha = -2$

$\therefore$ Coordinate of P is $(-24, 7)$

Also, $2X$ plane divides PQ in ratio 4 : 5.

The intersecting point is

$\left(\frac{12+5a}{9}, \frac{4\beta+20}{9}, \frac{32+35}{9}\right)$

On $2x$ plane, y-coordinate of any point is 0 .

$\Rightarrow \quad 4\beta + 20 = 0 \Rightarrow \beta = -5$

$\therefore$ Coordinate of Q is $(3, -5.8)$

$$\begin{aligned} PQ &= \sqrt{13+2)^2 + (-5-4)^2 + (8-7)^2} \\ &= \sqrt{25+81+1} \\ &= \sqrt{107} \end{aligned}$$

Question67

If the distance between the planes $2x + y + z + 1 = 0$ and $2x + y + z + \alpha = 0$ is 3 units, then product of all possible values of α is

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Options:

A. -43

B. 43

C. 53

D. -53

Answer: D

Solution:

The planes

$$2x + y + z + 1 = 0 \quad \text{and} \quad 2x + y + z + \alpha = 0$$

are parallel (same normal $(2, 1, 1)$).

Distance between parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is

$$\frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}.$$

So here:

$$\frac{|\alpha - 1|}{\sqrt{2^2 + 1^2 + 1^2}} = \frac{|\alpha - 1|}{\sqrt{6}} = 3 \Rightarrow |\alpha - 1| = 3\sqrt{6}$$

$$\Rightarrow \alpha = 1 \pm 3\sqrt{6}.$$

Product of all possible values:

$$(1 + 3\sqrt{6})(1 - 3\sqrt{6}) = 1 - (3\sqrt{6})^2 = 1 - 54 = -53.$$

$$\boxed{-53}$$

Question68

If P divides the line segment joining the points $A(1, 2, -1)$ and $B(-1, 0, 1)$ externally in the ratio 1 : 2 and $Q = (1, 3, -1)$, then $PQ =$

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Options:

A. $\sqrt{10}$

B. 3

C. 1

D. $\sqrt{13}$

Answer: B

Solution:

Given, points $A(1,2,-1)$ and $B(-1,0,1)$ and P divides the line segment externally in the ratio $1:2$.

For external division, coordinates are

$$\left[\frac{m_1x_2-m_2x_1}{m_1-m_2}, \frac{m_1y_2-m_2y_1}{m_1-m_2}, \frac{m_1z_2-m_2z_1}{m_1-m_2}\right]$$
$$=\left[\frac{1(-1)-2(1)}{1-2}, \frac{1(0)-2(2)}{1-2}, \frac{1(1)-2(-1)}{1-2}\right]$$
$$=\left[\frac{-1-2}{-1}, \frac{-4}{-1}, \frac{3}{-1}\right]$$
$$=[3,4,-3]$$

So, $PQ=\sqrt{(3-1)^2+(4-3)^2+(-3+1)^2}$
$=\sqrt{(2)^2+(1)^2+(-2)^2}$
$=\sqrt{4+1+4}=3$ units

Question69

If the direction cosines of a line are $\left(\frac{a}{\sqrt{83}}, \frac{5}{\sqrt{83}}, \frac{c}{\sqrt{83}}\right)$ and $c-a=4$, then $ca=$

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Options:

A. 24

B. 21

C. 18

D. 33

Answer: B

Solution:

Direction cosine of $a\hat{i}+b\hat{j}+c\hat{k}$ can be $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$

According to question,

$\sqrt{a^2+b^2+c^2}=\sqrt{83}, b=5$

$\Rightarrow \quad a^2+(5)^2+c^2=83$
$\Rightarrow \quad a^2+c^2=83-25$
$\Rightarrow \quad a^2+c^2=58 \quad \ldots$ (i)

and $c-a=4$ [given]

$(c-a)^2+2ac=a^2+c^2$
$\Rightarrow (4)^2+2ac=58$ [from Eq. (i), $c^2+a^2=58$]
$\Rightarrow 2ac=58-16$
$ac=\frac{42}{2}=21$

Question70

Let the plane π pass through the point (1, 0, 1) and perpendicular to the planes $2x+3y-z=2$ and $x-y+2z=1$. Let the equation of the plane passing through the point (11, 7, 5) and parallel to the plane π be $ax+by-z-d=0$. Then, $\frac{a}{b}+\frac{b}{d}=$

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Options:

A. 3

B. 0

C. 2

D. -2

Answer: D

Solution:

The equation of plane passes through the point $(1,0,1)$ is

$a(x-1)+b(y-0)+c(z-1)=0 \quad \ldots$ (i)

If this plane is perpendicular to the planes $2x+3y-z=2$ and $x-y+2z=1$

Then, $2a+3b-c=2$ and $a-b+2c=1$

$$\Rightarrow \frac{a}{6-1}=\frac{b}{-1-4}=\frac{c}{2(-1)-3}$$
$$\Rightarrow \frac{a}{5}=\frac{b}{-3}=\frac{c}{-5}=k$$

Let $a=5k, b=-3k, c=-5k$

Putting the values of a, b and c in Eq. (i), we get the required equation of plane

$$\begin{aligned} & 5k(x-1)+(-3k)(y)+(-5k)(z-1)=0 \\ \Rightarrow & 5x-5-3y-5z+5=0 \\ \Rightarrow & 5x-3y-5z=0 \end{aligned}$$

Now, plane passing through $(11,7,5)$ and parallel to the plane $\pi(5x-3y-5z)$.

Equation of a plane parallel to

$5x-3y-5z=k$ ... (i)

and passing through $(11,7,5)$

$$\begin{aligned} & 5(11)-3(7)-5(5)=k \\ \Rightarrow & 55-21-25=k \\ \Rightarrow & k=9 \end{aligned}$$

So, equation of plane

$\Rightarrow 5x-3y-5z=9$

Comparing with $ax+by-z+d=0$ (given)

So, $a=5, b=-3, c=5, d=9$

$$\frac{a}{b}+\frac{b}{d}=\frac{5}{-3}+\frac{-3}{9}$$
$$\Rightarrow \frac{a}{b}+\frac{b}{d}=\frac{-15-3}{9}=\frac{-18}{9}=-2$$

Question71

D, E, F are respectively the points on the sides BC, CA and AB of a $\triangle ABC$ dividing them in the ratio $2:3, 1:2, 3:1$ internally. The lines BE and CF intersect on the line AD at P. If $\mathbf{AP}=x_1 \cdot \mathbf{AB}+y_1 \cdot \mathbf{AC}$, then $x_1+y_1=$

Options:

A. 5/6

B. 1

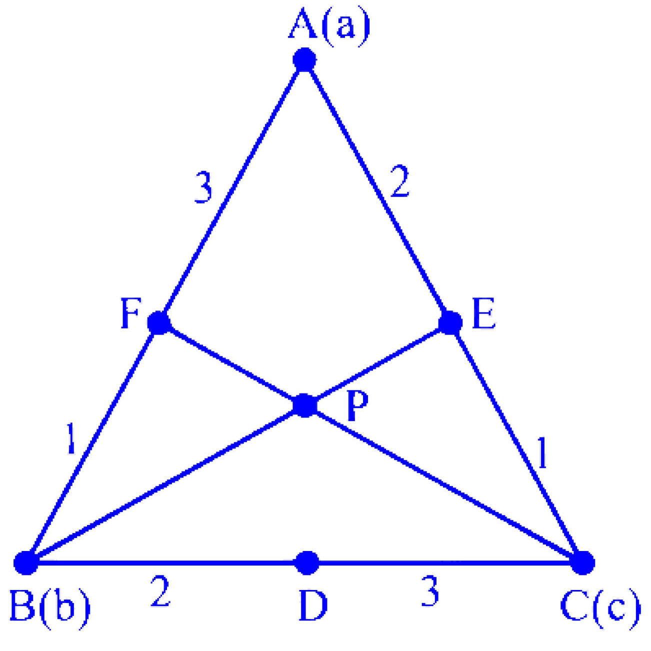
C. 3/2

D. 2

Answer: A

Solution:

Let a, b and c are the position vectors of the vertices A, B are C, respectively.



Also $\mathbf{d}, \mathbf{e}$ and $\mathbf{f}$ be the position vectors of points D, E and F, respectively

$\mathrm{d}=\frac{2 \mathrm{c}+3 \mathrm{~b}}{5}, \mathrm{e}=\frac{2 \mathrm{c}+\mathrm{a}}{3}, \mathrm{f}=\frac{3 \mathrm{~b}+\mathrm{a}}{4}$

Equation of line BE is $\mathbf{r}=\mathbf{b}+k(\mathbf{e}-\mathbf{b})$,

$$\mathbf{r}=\mathbf{b}+k\left(\frac{2 \mathbf{c}+\mathbf{a}}{3}-\mathbf{b}\right)$$

$$\Rightarrow \quad \mathbf{r}=\mathbf{b}+k\left[\frac{2 \mathbf{c}+\mathbf{a}-3 \mathbf{b}}{3}\right]$$

$$\Rightarrow \quad \mathbf{r}=\frac{k}{3} \mathbf{a}+\mathbf{b}[1-k]+\frac{2 k}{3} \mathbf{c}$$

Equation of line CF is

$$\begin{aligned} \mathbf{r} &= \mathbf{c} + \lambda(\mathbf{f} - \mathbf{c}) \\ &= \mathbf{c} + \lambda\left(\frac{3b + \mathbf{a}}{4} - \mathbf{c}\right) = \mathbf{c} + \lambda\left[\frac{3b + \mathbf{a} - 4\mathbf{c}}{4}\right] \\ &= \frac{\lambda \mathbf{a}}{4} + \frac{3\lambda \mathbf{b}}{4} + \mathbf{c}(1 - \lambda) \end{aligned}$$

CF and BE intersect at point P

$\frac{k\mathbf{a}}{3} + \mathbf{b}(1-k) + \frac{2k}{3}\mathbf{c} = \frac{\lambda \mathbf{a}}{4} + \frac{3\lambda \mathbf{b}}{4} + \mathbf{c}(1-\lambda)$

On comparing,

$\frac{k}{3} = \frac{\lambda}{4}$

$\Rightarrow \quad k = \frac{3\lambda}{4}$

Also,

$$\begin{aligned} & 1-k &&= \tfrac{3\lambda}{4} \Rightarrow 1-k = k \\ \Rightarrow \quad & 2k &&= 1 \Rightarrow k = \tfrac{1}{2} \\ \text{and} \quad & \tfrac{1}{2} &&= \tfrac{3\lambda}{4} \\ & \lambda &&= \tfrac{2}{3} \end{aligned}$$

Position vector of $P = \frac{\mathbf{a}}{6} + \frac{\mathbf{b}}{2} + \frac{\mathbf{c}}{3} = \frac{\mathbf{a}+3\mathbf{b}+2\mathbf{c}}{6}$

$$\begin{aligned} &\mathbf{AP} = x_1 \mathbf{AB} + y_1 \mathbf{AC} \\ &\left(\frac{\mathbf{a} + 3\mathbf{b} + 2\mathbf{c}}{6} - \mathbf{a}\right) = x_1(\mathbf{b} - \mathbf{a}) + y_1(\mathbf{c} - \mathbf{a}) \\ &\Rightarrow \frac{-5\mathbf{a} + 3\mathbf{b} + 2\mathbf{c}}{6} = \mathbf{a}\left(-x_1 - y_1\right) + x_1 \mathbf{b} + y_1 \mathbf{C} \\ &\therefore \quad -x_1 - y_1 = \frac{-5}{6} \\ &x_1 + y_1 = \frac{5}{6} \end{aligned}$$

Question72

If the equation of the plane passing through the point $A(-2, 1, 3)$ and perpendicular to the vector $3\hat{i} + \hat{j} + 5\hat{k}$ is $ax + by + cz + d = 0$, then $\frac{a+b}{c+d} =$

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Options:

A. 4/5

B. 2/3

C. 1

D. $-4/5$

Answer: D

Solution:

The equation of the plane passing through the point $A(-2, 1, 3)$ and perpendicular to the vector $3\hat{i} + \hat{j} + 5\hat{k}$ is $3x + y + 5z = d$.

Thus, $d = -6 + 1 + 15 = 10$

Then, $3x + y + 5z - 10 = 0$

Thus, $a = 3, b = 1, c = 5, d = -10$

$\frac{a+b}{c+d} = \frac{3+1}{5-10} = -\frac{4}{5}$

Question73

If x-coordinate of a point P on the line joining the points $Q(2, 2, 1)$ and $R(5, 2, -2)$ is 4, then the y-coordinate of $P =$

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Options:

A. $-\frac{1}{2}$ (x-coordinate of P)

B. -2 (z-coordinate of P)

C. 2 (z-coordinate of P)

D. Sum of x and z coordinates of P

Answer: B

Solution:

Given, $Q(2,2,1)$ and $R(5,2,-2)$.

Thus, equation of line through R and Q is

$$\frac{x-2}{2-5}=\frac{y-2}{2-2}=\frac{z-1}{1+2}$$
$$\Rightarrow \frac{x-2}{-3}=\frac{y-2}{0}=\frac{z-1}{3}=r \text{ (say)}$$

Thus, P be point on the line. So, $P=(-3r+2), 2,(3r+1)$

Since, x-coordinate is 4 . So, $-3r+2=4$

$\Rightarrow \quad r=\frac{-2}{3}$

Thus, y-coordinate $=2$

and z-coordinate $=-1$

$\therefore$ y-coordinate $=-2$(z - coordinate of P)

Question74

If $(2,3,c)$ are the direction ratios of a ray passing through the point $C(5,q,1)$ and also the mid-point of the line segment joining the points $A(p,-4,2)$ and $B(3,2,-4)$, then $c \cdot (p+7q)=$

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Options:

A. 17

B. 34

C. 21

D. 28

Answer: B

Solution:

Mid-point of A and B is $\left(\frac{p+3}{2}, \frac{-4+2}{2}, \frac{2-4}{2}\right)=\left(\frac{p+3}{2},-1,-1\right)$

Let P be mid-point of A and B.

DR'S of $CP=\frac{p+3}{2}-5,-1-q,-1-1$

$=\frac{p-7}{2},-1-q,-2$

$$\text{Here, } \frac{p-7}{2}=2,-1-q=3, c=-2$$
$$\Rightarrow \quad p=11, q=-4, c=-2$$
$$\Rightarrow \quad c(p+7q)=-2[11-28]=-2(-17)=34$$

Question75

If the equation of the plane which is at a distance of $1/3$ units from the origin and perpendicular to a line whose directional ratios are $(1,2,2)$ is $x+py+qz+r=0$, then $\sqrt{p^2+q^2+r^2}=$

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Options:

A. 3

B. $\sqrt{5}$

C. $\sqrt{13}$

D. 2

Answer: A

Solution:

Given, the plane normal to the vector

$\hat{n}=\hat{i}+2\hat{j}+2\hat{k}$

Let equation of plane

$x+2y+2z+d=0$

Distance of a plane from the origin is $1/3$.

Thus, distance $= \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

$$\Rightarrow \frac{1}{3} = \left|\frac{0+2\cdot 0+2\cdot 0+d}{\sqrt{1+2^2+2^2}}\right|$$
$$\Rightarrow 1/3 = d/3 \Rightarrow d = 1$$

Thus, equation of the plane is $x+2y+2z+1=0$

Comparing given equation of the plane

$$x+py+qz+r=0$$
$$\therefore \quad p=2, q=2, r=1$$
$$\therefore \quad \sqrt{p^2+q^2+r^2} = \sqrt{2^2+2^2+1^2}$$
$$\Rightarrow \quad \sqrt{9} = 3$$

Question76

The point of intersection of the lines $\mathbf{r} = 2\mathbf{b} + t(6\mathbf{c} - \mathbf{a})$ and $\mathbf{r} = \mathbf{a} + s(\mathbf{b} - 3\mathbf{c})$ is

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Options:

A. $a+b+c$

B. $\mathrm{b}-\mathrm{c}-6\mathrm{a}$

C. $2a-b+c$

D. $a+2b-6c$

Answer: D

Solution:

$\mathbf{r} = 2\mathbf{b} + t(6\mathbf{c} - \mathbf{a})$

and $\mathbf{r} = \mathbf{a} + s(\mathbf{b} - 3c)$

Put $t=-1$ and $s=24$ and check

If $t=-1 \Rightarrow \mathbf{r} = 2\,\mathrm{b} - 1(6\mathrm{c} - \mathrm{a}) = 2\,\mathrm{b} + \mathrm{a} - 6\mathrm{c}$

If $s = 2 \Rightarrow \mathbf{r} = \mathbf{a} + 2(\mathbf{b} - 3\mathrm{c}) = \mathbf{a} + 2\mathbf{b} - 6\mathbf{c}$

Therefore, we can say that $\mathbf{a} + 2b - 6c$ lies on both the lines, i.e. it is the intersecting point.

Question77

If the point $(a, 8, -2)$ divides the line segment joining the points $(1,4,6)$ and $(5,2,10)$ in the ratio $m : n$, then $\frac{2m}{n} - \frac{a}{3} =$

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Options:

A. -7

B. 1

C. -2

D. 3

Answer: B

Solution:

$$x = \frac{mx_2 + nx_1}{m+n}$$
$$\Rightarrow \quad a = \frac{5m+n}{m+n}$$
$$\Rightarrow \quad a = \frac{5\left(\frac{m}{n}\right)+1}{\frac{m}{n}+1} \quad \text{.... (i)}$$

y-coordinate $: 8 = \dfrac{2m+4n}{m+n}$

$$\Rightarrow \quad 8m + 8n = 2m + 4n$$
$$\Rightarrow \quad 6m = -4n$$
$$\Rightarrow \quad 3m = -2n \quad \text{... (ii)}$$

z - coordinate $: -2 = \frac{10m+6n}{m+n}$

$$\Rightarrow -2m - 2n = 10m + 6n$$
$$\Rightarrow -8n = 12m \Rightarrow -2n = 3m$$
$$\Rightarrow \frac{m}{n} = -\frac{2}{3} \quad \text{... (iii)}$$

From Eq. (iii), put $\frac{m}{n} = \frac{-2}{3}$ into Eq. (i),

$$a = \frac{5\left(-\frac{2}{3}\right)+1}{-\frac{2}{3}+1} = \frac{-10+3}{-2+3} = -7$$

$$\therefore \quad \frac{2m}{n} - \frac{a}{3} = 2\left(-\frac{2}{3}\right) - \left(-\frac{7}{3}\right)$$

$$= \frac{-4}{3} + \frac{7}{3} = \frac{-4+7}{3} = \frac{3}{3} = 1$$

Question78

If (a, b, c) are the direction ratios of a line joining the points $(4, 3, -5)$ and $(-2, 1, -8)$, then the point $P(a, 3b, 2c)$ lies on the plane

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Options:

A. $x + y + z = 0$

B. $x + y - 2z = 0$

C. $x + 2y + 3z = 0$

D. $x - 2y + 3z = 0$

Answer: B

Solution:

Given, points are $(4, 3, -5)$ and $(-2, 1, -8)$.

Direction ratios

$$(a, b, c) = (4 - (-2), 3 - 1, -5 - (-8)) = (6, 2, 3)$$
$$\therefore \quad P(a, 3b, 2c) \equiv (6, 3 \times 2, 2 \times 3) \equiv (6, 6, 6)$$
For $x + y - 2z = 0$
$$\Rightarrow \quad 6 + 6 - 2 \times 6 = 0$$
$$\Rightarrow \quad 12 - 12 = 0 \text{ (true)}$$

Question79

The x-intercept of a plane π passing through the point $(1,1,1)$ is $\frac{5}{2}$ and the perpendicular distance from the origin to the plane π is $\frac{5}{7}$. If the y-intercept of the plane π is negative and the z-intercept is positive, then its y-intercept is

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Options:

A. $-5/3$

B. $-5/6$

C. $-3/2$

D. $-5/2$

Answer: A

Solution:

Let plane's equation be

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad \text{....(i)}$$

$$\Rightarrow \quad a=\frac{5}{2}$$

and plane passes through $(1,1,1)$.

$$\therefore \quad \frac{1}{5/2}+\frac{1}{b}+\frac{1}{c}=1$$

$$\Rightarrow \quad \frac{1}{b}+\frac{1}{c}=1-\frac{2}{5}$$

$$\Rightarrow \quad \frac{1}{b}+\frac{1}{c}=\frac{3}{5} \quad \text{.... (ii)}$$

Perpendicular distance of plane from origin $=\frac{5}{7}$

$$\left|\frac{0+0+0-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right|=\frac{5}{7}$$

$$\Rightarrow \quad \left|\frac{-1}{\sqrt{\left(\frac{2}{5}\right)^2+\frac{1}{b^2}+\frac{1}{c^2}}}\right|=\frac{5}{7}$$

$$\Rightarrow \quad (-1)^2\left(\frac{7}{5}\right)^2=\left(\frac{2}{5}\right)^2+\frac{1}{b^2}+\frac{1}{c^2}$$

$$\Rightarrow \quad \frac{49}{25}-\frac{4}{25}=\frac{1}{b^2}+\frac{1}{c^2}$$

$$\Rightarrow \quad \frac{1}{b^2}+\frac{1}{c^2}=\frac{45}{25}=\frac{9}{5} \quad \text{... (iii)}$$

On squaring Eq. (ii), we get

$$\frac{1}{b^2}+\frac{1}{c^2}+\frac{2}{bc}=\frac{9}{25}$$

$$\Rightarrow \frac{9}{5}+\frac{2}{bc}=\frac{9}{25} \quad \text{[from Eq. (iii)]}$$

$$\Rightarrow \frac{2}{bc}=\frac{-36}{25}$$

$$\Rightarrow bc=-\frac{25}{18}$$

Putting $c=-\frac{25}{18}\cdot\frac{1}{b}$ into Eq. (ii), we get

$$\frac{1}{b}+\frac{1}{\left(\frac{-25}{18}\cdot\frac{1}{b}\right)}=\frac{3}{5}$$

$$\Rightarrow \quad \frac{1}{b}-\frac{18b}{25}=\frac{3}{5}$$

$$\Rightarrow \quad 25-18b^2=15b$$

$$\Rightarrow \quad 18b^2+15b-25=0$$

$$\Rightarrow \quad 18b^2+30b-15b-25=0$$

$$\Rightarrow \quad 6b(3b+5)-5(3b+5)=0$$

$$\Rightarrow \quad (3b+5)(6b-5)=0$$

$$\Rightarrow \quad b=-\frac{5}{3},\frac{5}{6}$$

$\because$ y-intercept is negative.

$b=-\frac{5}{3}$

Question80

The equation of the plane passing through $3\hat{i}+2\hat{j}+6\hat{k}$ and parallel to the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is

Options:

A. $x+y+z=11$

B. $2x-y-3z=-14$

C. $2x-y+z=10$

D. $x-2y+3z=17$

Answer: B

Solution:

Given that, equation of plane parallel to vectors $2\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$. Also, it passes through $3\hat{\mathbf{i}}+2\hat{\mathbf{j}}+6\hat{\mathbf{k}}$.

Let the plane be

$a(x-3)+b(y-2)+c(z-6)=0$ ...... (i)

such that,

$\mathrm{t}a(2)+b(1)+c(1)=0$

$\Rightarrow 2a+b+c=0$ .... (ii)

and $a(1)+b(-1)+c(1)=0$

$\Rightarrow \quad a-b+c=0$ .... (iii)

From Eq. (ii) and (iii)

$\Rightarrow \quad \frac{a}{2}=\frac{b}{-1}=\frac{c}{-3}=\lambda \quad$ (say)

$\Rightarrow a=2\lambda, b=-\lambda, c=-3\lambda$

From Eq. (i), we get

$$\begin{aligned} & 2\lambda(x-3)-\lambda(y-2)-3\lambda(z-6)=0 \\ \Rightarrow \quad & 2(x-3)-(y-2)-3(z-6)=0 \\ \Rightarrow \quad & 2x-y-3z=-14 \end{aligned}$$

Question81

The direction cosines of the line joining the points $(-2,4,-5)$ and $(1,2,3)$ are

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Options:

A. $\left(\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}\right)$

B. $\left(\frac{3}{\sqrt{77}}, \frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}}\right)$

C. $(1,0,0)$

D. $\left(\frac{-3}{77}, \frac{-2}{77}, \frac{8}{77}\right)$

Answer: A

Solution:

Given point, $A(-2,4,-5)$ and $B(1,2,3)$

$AB = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

DR's are $(3,-2,8)$

Then, direction cosines $=(l, m, n)$

where, $l = \frac{3}{\sqrt{3^2+(-2)^2+8^2}} = \frac{3}{\sqrt{77}}$

$m = \frac{(-2)}{\sqrt{77}}$ and $n = \frac{8}{\sqrt{77}}$

$\therefore$ DC's are $\left(\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}\right)$

Question82

The points (2, 3, 4), (−1, −2, 1) and (5, 8, 7) are

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Options:

A. collinear

B. vertices of a right angled triangle

C. vertices of a equilateral triangle

D. vertices of an isosceles triangle

Answer: A

Solution:

Given, points $(2,3,4),(-1,-2,1)$ and $(5,8,7)$

If $\Delta=0$, then points are collinear.

$\Delta \neq 0$, then it represent vertices of triangle

$$\Delta=\begin{vmatrix} 2 & 3 & 4 \\ -1 & -2 & 1 \\ 5 & 8 & 7 \end{vmatrix}$$
$$=2(-14-8)-3(-7-5)+4(-8+10)$$
$$=-44+36+8=0$$

$\Rightarrow$ Given points are collinear.

Question83

The sum of intercepts of the plane $4x+3y+2z=2$ on the coordinate axes is

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Options:

A. $\frac{13}{6}$

B. 9

C. $\frac{13}{12}$

D. 2

Answer: A

Solution:

Given equation of plane is

$4x + 3y + 2z = 2$ .... (i)

General form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where a, b and c are intercept on coordinate axis, convert Eq. (i) in general form as

$\frac{4}{2}x + \frac{3}{2}y + \frac{2}{2}z = 1$

or $\quad 2x + \frac{3}{2}y + z = 1 \Rightarrow \frac{x}{1/2} + \frac{y}{2/3} + \frac{z}{1} = 1$

$$\begin{aligned} \therefore x\text{-intercept } &= \frac{1}{2} = (a) \\ y\text{-intercept } &= \frac{2}{3} = (b) \\ z\text{-intercept } &= 1 = (c) \end{aligned}$$

Then, $a + b + c = \frac{1}{2} + \frac{2}{3} + 1 = \frac{3+4+6}{6} = \frac{13}{6}$

Question84

If the lines, $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$ are coplanar, then $\sin^{-1}(\sin\lambda) + \cos^{-1}(\cos\lambda)$ is equal to

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Options:

A. $8 - 2\pi$

B. $6 - \pi$

C. $3\pi - 8$

D. $4\pi - 8$

Answer: C

Solution:

Since we know that two lines are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \text{ .... (i)}$$

Given lines are

$$L_1 : \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{6}$$
$$\therefore \quad x_1 = 3, y_1 = 2, z_1 = 1$$
$$a_1 = 2, b_1 = 3, c_1 = \lambda$$
$$L_2 : \frac{x-z}{3} = \frac{y-3}{2} = \frac{z-2}{3}$$
$$a_2 = 3, b_2 = 2, c_2 = 3$$

From Eq. (i)

$$\begin{vmatrix} 1 & -1 & -1 \\ 2 & 3 & \lambda \\ 3 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 2 & 5 & \lambda + 2 \\ 3 & 5 & 6 \end{vmatrix} = 0$$
$$\Rightarrow \quad 30 - 5\lambda - 10 = 0$$
$$\Rightarrow \quad \lambda = 4$$
$$\Rightarrow \sin^{-1}(\sin 4) + \cos^{-1}(\cos 4)$$
$$(\pi - 4) + (2\pi - 4) = (3\pi - 8)$$

Question85

The line passing through $(1, 1, -1)$ and parallel to the vector $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ meets the line $\frac{x-3}{-1} = \frac{y+2}{5} = \frac{z-2}{-4}$ at A and the plane $2x - y + 2z + 7 = 0$ at B. Then AB is equal to

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Options:

A. $\sqrt{6}$

B. $2\sqrt{6}$

C. $3\sqrt{6}$

D. $4\sqrt{6}$

Answer: B

Solution:

Line passing through $(1,1,-1)$ and parallel to $(1,2,-1)$ is

$$L_1 : \frac{x-1}{1} = \frac{y-1}{2} = \frac{z+1}{-1} = \lambda$$
$$L_2 : \frac{x-3}{-1} = \frac{y+2}{5} = \frac{z-2}{-4} = \propto$$

For $A : (\lambda+1, 2\lambda+1, -\lambda-1)$

$$= (-\propto +3, 5\propto -2, -4\propto +2)$$
$$\Rightarrow \quad \lambda + \propto = 2 \Rightarrow 2\lambda + 1 = 5\propto -2$$
$$\Rightarrow \quad 4 - 2\propto +1 = 5\propto -2$$
$$1 = \propto \quad \text{and } \lambda = 1$$
$$A : (2,3,-2)$$
$$P : 2x - y + 2z + 7 = 0$$
$$\Rightarrow 2(\lambda+1) - (2\lambda+1) + (-2\lambda-2) + 7 = 0$$
$$\Rightarrow \quad -2\lambda + 6 = 0 \Rightarrow \lambda = 3$$
$$B : (4,7,-4)$$
$$AB^2 = 2^2 + 4^2 + 2^2 = 24 = 2\sqrt{6}$$

Question86

If the vertices of the triangles are (1, 2, 3), (2, 3, 1), (3, 1, 2) and if H, G, S and I respectively denote its orthocentre, centroid, circumcentre and incentre, then H + G + S + I is equal to

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Options:

A. (2, 2, 2)

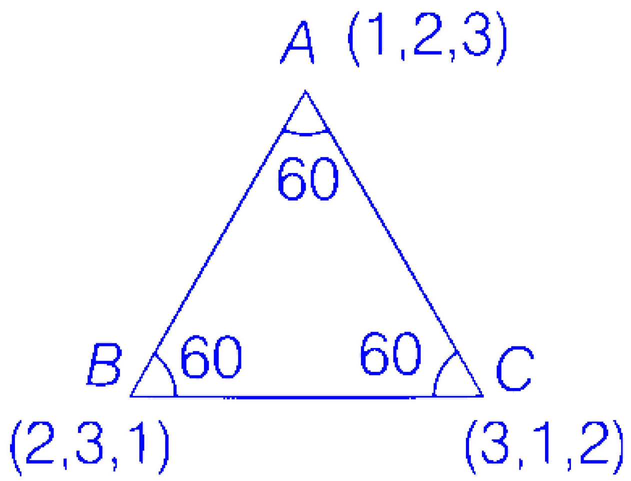
B. (4, 4, 4)

C. (6, 6, 6)

D. (8, 8, 8)

Answer: D

Solution:



Centroid = (2, 2, 2)

∵ Triangle is equilateral

(2, 2, 2) = H = G = S = I

∴ H + G + S + I = (8, 8, 8)

Question87

A(2, 3, 4), B(4, 5, 7), C(2, −6, 3) and D(4, −4, k) are four points. If the line AB is parallel to CD, then k is equal to

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Options:

A. 2

B. 4

C. 5

D. 6

Answer: D

Solution:

AB = λCD

$\Rightarrow (2,2,3) = \lambda(2,2,k-3)$

$\Rightarrow k-3=3 \Rightarrow k=6$

Question88

If the direction cosines of two lines are $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$ and $\left(\frac{5}{13}, \frac{12}{13}, 0\right)$, then identify the direction ratios of a line which is bisecting one o the angle between them.

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Options:

A. (40, 60, 13)

B. (41, 60, 10)

C. (41, 62, 13)

D. (1, 2, 3)

Answer: C

Solution:

P = Angle bisector

$= \frac{1}{2}\left[\left(\frac{2}{3} \pm \frac{5}{13}\right), \left(\frac{2}{3} \pm \frac{12}{13}\right), \left(\frac{1}{3} \pm 0\right)\right] = \frac{1}{2}\left(\frac{41}{39}, \frac{62}{39}, \frac{1}{3}\right)$

Parallel to this vector will be (41, 62, 13).

Question89

X intercept of the plane containing the line of intersection of the planes $x-2y+z+2=0$ and $3x-y-z+1=0$ and also passing through $(1,1,1)$ is

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Options:

A. $\frac{1}{3}$

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: C

Solution:

Equation of plane containing line ofintersection of the given planes is
$(x-2y+z+2)+\lambda(3x-y-z+1)=0$

This plane also passes through (1, 1, 1).

$\therefore(1-2+1+2)+\lambda(3-1-1+1)=0$

$\Rightarrow\lambda=-1$

$\therefore$ Required plane $2x+y-2z-1=0$

x-intercept of these plane is (x, 0, 0)

$\Rightarrow 2x-1=0$

$x=\frac{1}{2}$

Question90

Let L_1 (resp, L_2) be the line passing through $2\hat{\mathbf{i}}-\hat{\mathbf{k}}$ (resp. $2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}$) and parallel to $3\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ (resp. $\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$). Then the shortest distance between the lines L_1 and L_2 is equal to

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Options:

A. $\frac{10}{\sqrt{35}}$

B. $\frac{8}{\sqrt{35}}$

C. $\frac{11}{\sqrt{35}}$

D. $\frac{9}{\sqrt{35}}$

Answer: D

Solution:

Given that line L_1 is passing through $(2\hat{\mathbf{i}}-\hat{\mathbf{k}})$ and parallel to $(3\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}})$

Let $\mathbf{a}_1=2\hat{\mathbf{i}}-\hat{\mathbf{k}}$ and $\mathbf{b}_1=3\hat{\mathbf{i}}-\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

equation line $L_1:\mathbf{r}=\mathbf{a}_1+\lambda\mathbf{b}_1$ ..... (i)

Also given line L_2 is passing through $(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}})$ and parallel to $(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}})$

Let $\mathbf{a}_2=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}$ and $\hat{\mathbf{b}}_2=\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$

Equation of line $L_2:\mathbf{r}=\mathbf{a}_2+\mathbf{b}_2$ ..... (ii)

$\therefore$ Shortest distance between L_1 and L_2

$d=\left|\frac{(\mathbf{b}_1\times\mathbf{b}_2)\cdot(\mathbf{a}_2-\mathbf{a}_1)}{|\mathbf{b}_1\times\mathbf{b}_2|}\right|$ ..... (iii)

Consider $\mathbf{b}_1\times\mathbf{b}_2=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & -1 & 2\\ 1 & -2 & 1\end{vmatrix}=3\hat{\mathbf{i}}-\hat{\mathbf{j}}-5\hat{\mathbf{k}}$

$|\mathbf{b}_1\times\mathbf{b}_2|=\sqrt{3^2+(-1)^2+(-5)^2}=\sqrt{35}$

and $\quad \mathbf{a}_2-\mathbf{a}_1=(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}})-(2\hat{\mathbf{i}}-\hat{\mathbf{k}})$

$=\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

$$\begin{aligned}(\mathbf{b}_1\times\mathbf{b}_2)\cdot(\mathbf{a}_2-\mathbf{a}_1)&=(3\hat{\mathbf{i}}-\hat{\mathbf{j}}-5\hat{\mathbf{k}})\cdot(\hat{\mathbf{j}}-2\hat{\mathbf{k}})\\&=-1+10=9\end{aligned}$$

Now from Eq. (iii)

$$d = \left| \frac{9}{\sqrt{35}} \right|$$

$$\Rightarrow \quad d = \frac{9}{\sqrt{35}}$$

Question91

If the points $(2, 4, -1)$, $(3, 6, -1)$ and $(4, 5, -1)$ are three consecutive vertices of a parallelogram,then its fourth vertex is

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Options:

A. $(3, 3, 1)$

B. $(3, 1, 3)$

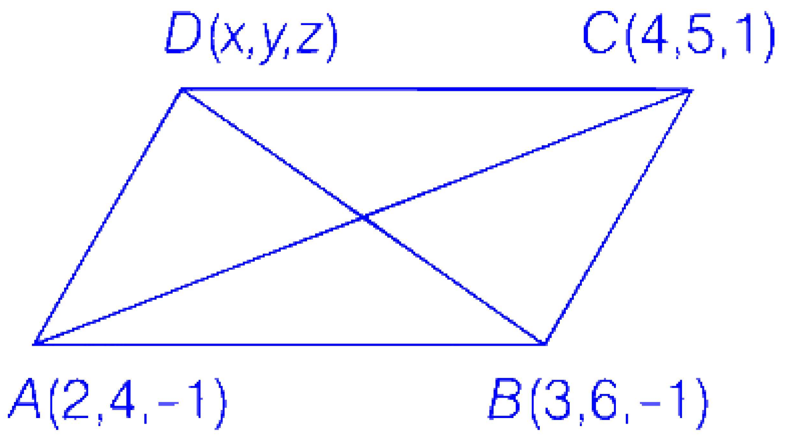
C. $(1, 3, 3)$

D. $(0, 0, 0)$

Answer: A

Solution:

In parallelogram, diagonals bisect each other



Let fourth vertex (x, y, z)

Mid-point of AC = mid-point of BD

$$\left(\frac{2+4}{2}, \frac{4+5}{2}, \frac{-1+1}{2}\right)=\left(\frac{x+3}{2}, \frac{y+6}{2}, \frac{z-1}{2}\right)$$
$$\Rightarrow \quad \left(3, \frac{9}{2}, 0\right)=\left(\frac{x+3}{2}, \frac{y+6}{2}, \frac{z-1}{2}\right)$$
$$\Rightarrow \quad \frac{x+3}{2}=3, \frac{y+6}{2}=\frac{9}{2}, \frac{z-1}{2}=0$$
$$\Rightarrow \quad x=3, y=3, z=1$$

Fourth vertex $(3,3,1)$

Question92

$A(-1,2-3), B(5,0,-6)$ and $C(0,4,-1)$ are the vertices of a $\triangle ABC$. The direction cosines of internal bisector of $\angle BAC$ are

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Options:

A. $\frac{25}{\sqrt{714}}, \frac{8}{\sqrt{714}}, \frac{-5}{\sqrt{714}}$

B. $\frac{25}{\sqrt{714}}, \frac{8}{\sqrt{714}}, \frac{5}{\sqrt{714}}$

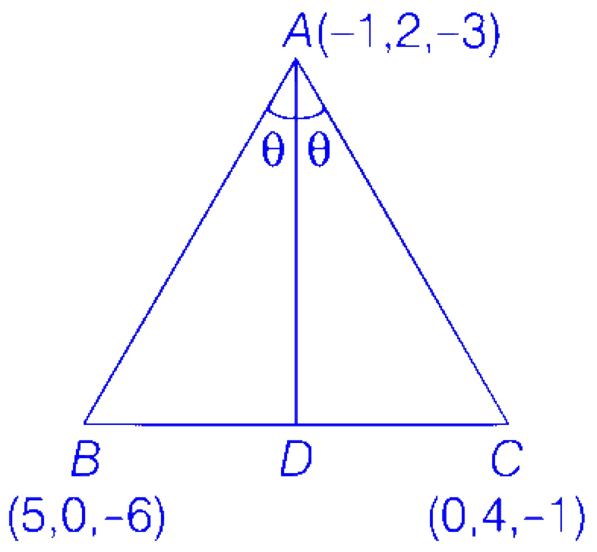
C. $\frac{5}{\sqrt{74}}, \frac{6}{\sqrt{74}}, \frac{8}{\sqrt{74}}$

D. $\frac{-5}{\sqrt{74}}, \frac{6}{\sqrt{74}}, \frac{-8}{\sqrt{74}}$

Answer: B

Solution:

$\mathbf{AB}=\mathbf{a}=\mathbf{B}-\mathbf{A}$



$= (5\hat{i} - 6\widehat{k}) - (-\hat{i} + 2\hat{j} - 3\widehat{k})$

$= 6\hat{i} - 2\hat{j} - 3\widehat{k}$

$AC = c = C - A$

$= (4\hat{j} - \widehat{k}) - (-\hat{i} + 2\hat{j} - 3\widehat{k})$

$= \hat{i} + 2\hat{j} + 2\widehat{k}$

Internal angle bisector $= \hat{\mathbf{c}} + \hat{\mathbf{a}}$

$$= \frac{\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{1+4+4}} + \frac{6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{\sqrt{36+4+9}}$$

$$= \frac{\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{3} + \frac{6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{7}$$

$$= \frac{7\hat{\mathbf{i}} + 18\hat{\mathbf{i}}}{21} + \frac{14\hat{\mathbf{j}} - 6\hat{\mathbf{j}}}{21} + \frac{14\hat{\mathbf{k}} - 9\hat{\mathbf{k}}}{21}$$

$$= \frac{25\hat{\mathbf{i}}}{21} + \frac{8\hat{\mathbf{j}}}{21} + \frac{5\hat{\mathbf{k}}}{21}$$

DR's of internal bisector of

$\angle BAC = \frac{25}{21}, \frac{8}{21}, \frac{5}{21}$

Direction Cosines $= \frac{25}{\sqrt{714}}, \frac{8}{\sqrt{714}}, \frac{5}{\sqrt{714}}$

Question93

If the projections of the line segment AB on xy, yz and zx planes are $\sqrt{15}, \sqrt{46}, 7$ respectively, then the projection of AB on Y-axis is

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Options:

A. 9

B. 3

C. 4

D. 7

Answer: A

Solution:

Let $\overline{AB} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$

a, b and c are projections of $\overline{AB}$ on x, y and z respectively

Projection of $\overline{AB}$ on xy-plane

$= \sqrt{a^2 + b^2} = \sqrt{15}$

Projection of $\overline{AB}$ on yz-plane

$= \sqrt{b^2 + c^2} = \sqrt{46}$

Projection of $\overline{AB}$ on zx-plane

$= \sqrt{c^2 + a^2} = 7$

$\therefore \quad a^2 + b^2 = 15, b^2 + c^2 = 46, c^2 + a^2 = 49$

On solving $a = 3, b = \sqrt{6}, c = \sqrt{40}$

Projection of $\overline{AB}$ on Y-axis $= b = \sqrt{6}$

* no option matches *

Question94

Find the equation of the plane passing through the point $(2, 1, 3)$ and perpendicular to the planes $x - 2y + 2z + 3 = 0$ and $3x - 2y + 4z - 4 = 0$.

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Options:

A. $2x - y - 2z + 3 = 0$

B. $x - 2y + 2z - 3 = 0$

C. $2x - y + 2z - 3 = 0$

D. $2x + y - 2z - 3 = 0$

Answer: A

Solution:

Let Equation of plane passing through $(2, 1, 3)$ is

$l(x-2) + m(y-1) + n(z-3) = 0$ .... (i)

This plane is perpendicular to

$$\begin{aligned} & x - 2y + 2z + 3 = 0 \\ \text{and} \quad & 3x - 2y + 4z - 4 = 0 \\ \Rightarrow \quad & l - 2m + 2n = 0 \\ \text{and} \quad & 3l - 2m + 4n = 0 \\ \Rightarrow \quad & \frac{l}{-8+4} = \frac{m}{6-4} = \frac{n}{-2+6} \\ \Rightarrow \quad & \frac{l}{-4} = \frac{m}{2} = \frac{n}{4} \Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{2} \end{aligned}$$

From Eq. (i),

$$\begin{aligned} \therefore \quad & -2(x-2) + (y-1) - 2(z-3) = 0 \\ \Rightarrow \quad & -2x + 4 + y - 1 + 2z - 6 = 0 \\ \Rightarrow \quad & -2x + y + 2z - 3 = 0 \\ \Rightarrow \quad & 2x - y - 2z + 3 = 0 \end{aligned}$$

Question95

The ratio in which the YZ-plane divides the line joining (2, 4, 5) and (3, 5, −4) is

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Options:

A. 2 : 3 internally

B. 3 : 2 internally

C. 3 : 2 externally

D. 2 : 3 externally

Answer: D

Solution:

Let the line joining the points $A(2,4,5)$ and $B(3,5,-4)$ is divided by YZ-plane at $(0,y,z)$ in the ratio $m:n$. Then, by section formula $\frac{2n+3m}{m+n}=0$

$$\Rightarrow \quad \frac{m}{n}=-\frac{2}{3}$$
$$\Rightarrow \quad m:n=2:3$$

(externally)

Question96

The direction cosines of a line which makesequal angles with the coordinate axes are

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Options:

A. $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

B. $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

C. $\left(\frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}, \frac{\pm 1}{\sqrt{3}}\right)$

D. $\left(\frac{12}{15}, \frac{5}{13}, 0\right)$

Answer: C

Solution:

Let the line makes angle α, β, γ with the coordinate axes, then

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\alpha = \beta = \gamma \quad \text{(given)}$$
$$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$
or
$$3\cos^2\alpha = 1 \text{ or } \cos\alpha = \pm\frac{1}{\sqrt{3}}$$

Direction cosines are $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$

Question97

Let O be the origin and P be a point which is at a distance of 3 units from the origin. If the direction ratios of $\overline{OP}$ are $(1, -2, -2)$, then the coordinates of P are

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Options:

A. $(1, -2, -2)$

B. $(3, -6, -6)$

C. $\left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right)$

D. $\left(\frac{1}{9}, \frac{-2}{9}, \frac{-2}{9}\right)$

Answer: A

Solution:

Let $P(x, y, z)$ and $O(0,0,0)$

DR's of $OP = (x, y, z)$

But it is given that DR's of $OP = (1, -2, -2)$

$\because x = 1, y = -2, z = -2$

$\therefore$ Coordinate of $P = (1, -2, -2)$
